

# Final Exam Review

15-362 / 15-662 Computer Graphics

# Lecture 2: Linear Algebra and

- **How can we measure vectors?**
- **What is a Vector Space?**
- **Draw a geometric representation of each rule that vectors seem to obey.**
- **Can a function be a vector? Explain**
- **Add and scale vectors.**
- **Add and scale functions**
- **What is the norm of a vector?**
- **Associate each property of a norm with a geometric interpretation.**

# Lecture 2: Linear Algebra and

- Use the inner product for operations such as projection.
- Compute the inner product of functions.
- Give properties and an example of a linear map.
- Define span and basis..
- Compute an orthonormal basis from a set of vectors.
- Be able to use Gram-Schmidt orthonormalization.
- Know that orthonormalization of functions can be done by decomposing them into sinusoids.
- Be able to solve a simple system of linear equations, depict it geometrically, and represent it in matrix form.
- Be able to represent a linear map in matrix form.

# Lecture 2: Linear Algebra and

- **Euclidean norm is a notion of length preserved by rotations/translations/reflections of space. Be able to calculate it for a vector of any dimension.**
- **Compute the inner product of two n-dimensional vectors. What is the geometric meaning if an orthonormal basis is used?**
- **Compute the cross product of two three-dimensional vectors. What is the geometric meaning of this cross product? What is the geometric meaning of its magnitude?**
- **Use the cross product to do a quarter rotation of a vector within a plane.**
- **Represent the dot product using matrix notation.**
- **Represent the cross product using matrix notation.**

# Lecture 2: Linear Algebra and

- **Understand gradient as the best linear approximation and the direction of steepest ascent.**
- **When is the gradient not defined?**
- **Bonus: Be able to express gradients of simple matrix expressions**
- **What is a vector field? Give an example.**
- **Be able to compute divergence, curl, and the Laplacian of a vector field. Also be able to express the meaning of these terms geometrically, for example by drawing a diagram.**
- **What is the Hessian? Be able to compute the Hessian of a function.**

# Lecture 4: Transforms

- 1. Which of the following operations are linear transforms: scale, rotation, shear, translation, reflection, rotation about a point that is not the origin?**
- 2. Express scale as a linear transform**
- 3. Express rotation as a linear transform**
- 4. Express shear as a linear transform**
- 5. Express reflection as a linear transform**
- 6. Express translation as an affine transform**
- 7. Know what makes a transform linear vs. affine**
- 8. Know how to build transformation matrices from start and end configurations of your object**

# Lecture 4: Transforms

- **Create 2D and 3D transformation matrices to perform specific scale, shear, rotation, reflection, and translation operations**
- **Compose transformations to achieve compound effects**
- **Rotate an object about a fixed point**
- **Rotate an object about a given axis**
- **Create an orthonormal basis given a single vector**
- **Understand the equivalence of  $[x \ y \ 1]$  and  $[wx \ wy$**

# Lecture 4 (again): Perspective

- **Review:**
  - **Form an orthonormal basis**
  - **Create a rotation matrix to rotate any coordinate frame to xyz**
  - **Create the rotation matrix to rotate the xyz coordinate frame to any other frame**
  - **Know basic facts about rotation matrices / how to recognize a rotation matrix**
    - **Rows (also columns) are unit vectors**
    - **Rows (also columns) are orthogonal to one another**
    - **If our rows (or columns) are  $u$ ,  $v$ , and  $w$ , then**

# Lecture 4 (again): Perspective

- **Understand the motivation behind the projection matrix that projects the view frustum to a unit cube**
  - **Be able to draw / discuss the details of the view frustum**
  - **Prove that a standard projection matrix preserves some information about depth Write an algorithm for drawing lines that handles all edge cases (i.e., including edges that are exactly horizontal or vertical).**
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1. **How do we determine coverage for a triangle?**
  2. **Give 2 algorithms for determining triangle coverage**

# Lecture 6: Rasterization

- Textures are used for many things, beyond pasting images onto object surfaces. Be able to list and describe the use cases in this list:
  - Normal maps (create appearance of bumpy object on smooth surface by giving false normal to the lighting equations)
  - Displacement maps (encode offsets in the geometry of a surface, which is difficult to handle in a standard graphics pipeline)
  - (Bonus) Environment maps (store light information in all directions in a scene)
  - (Bonus) Ambient occlusion map (store exposure of geometry to ambient light for better representation of surface appearance with simple lighting models)
  - Can you think of / discover others?
- Know how to interpolate texture coordinates
- Know how to index into a texture and compute a correct color using bilinear interpolation
- Be able to create a mipmap and store it in memory
- Be able to compute color from multiple levels of mipmaps using trilinear interpolation
- What is the logic behind selecting an appropriate level in a mipmap?
- What can happen if we select a level that is too high resolution? too low resolution?

# Lecture 6: Rasterization

- 1. How should we choose the correct color for a pixel? There is not an exact right answer. However, you should be able to discuss some of the issues involved.**
- 2. What is aliasing, and what artifacts does it produce in our images and our animations? Give at least 3 different examples.**
- 3. One form of aliasing is where high frequencies masquerade as low frequencies. Give an example of this phenomenon.**
- 4. Suppose we have a single red triangle displayed against a blue background. Does this scene contain high frequencies?**
- 5. (Bonus) What does the Nyquist-Shannon theorem tell us about how image frequencies relate to required sampling rate?**
- 6. (Bonus) One practical solution on your graphics card for reducing aliasing (i.e., for antialiasing) is to take multiple samples per pixel and average to get pixel color. Try to use sampling theory to explain as precisely as you can why taking multiple samples per pixel can reduce aliasing artifacts**

# Lecture 7: Depth and Blending

- **Interpolate colors using barycentric coordinates**
- **Bonus: Compute barycentric coordinates of a point using implicit edge functions**
- **Compute barycentric coordinates of a point using triangle areas**
- **Estimate the location of a point inside a triangle given its barycentric coordinates**
- **Estimate the location of a point outside a triangle given its barycentric coordinates**
- **Estimate barycentric coordinates of a point from a drawing.**
- **Show that interpolation in 3D space followed by projection can**

# Lecture 7: Depth and Blending

- What is the depth buffer (Z-buffer) and how is it used for hidden surface removal?
- Where does the depth for each sample / fragment come from? Where is it computed in the graphics pipeline?
- Is the depth represented in the depth buffer the actual distance from the camera? If not, what is it?

$$C = \alpha_B B + (1 - \alpha_B) \alpha_A A$$

- What is the meaning of the alpha parameter in the [R G B a] color representation?
- Be able to use alpha to do compositing with the “Over” operator.
- Is “Over” commutative? If not, create a counterexample.
- What is premultiplied alpha, and how does it work?
- Be able to use premultiplied alpha for “Over” composition.

# Lecture 7: 3D Rotations (Bonus)

- **What is the problem of gimbal lock? Give an example where this problem occurs.**
- **How does using quaternions solve this problem?**
- **Know that every rotation can be expressed as rotation by some angle about some axis.**
- **Know how to go between quaternions and axis-angle format for rotations.**
- **Know that quaternions are expressed as higher dimensional complex numbers.**
- **Be able to work out quaternion multiplication**

# Lecture 9: Introduction to Geometry

- List some types of implicit surface representations
- What types of operations are easy with implicit surface representations?
- List some types of explicit surface representations
- What types of operations are easy with explicit surface representations?
- What is a manifold surface?
- Distinguish manifold from non-manifold surfaces
- Can a manifold surface have a boundary? Give an example.
- Describe how to store mesh information in vertex and face tables. Give an example. What are good and bad points of this data structure?
- How would you store a mesh using incidence matrices? What are good and bad points of this data structure?
- What do you need to store in a halfedge data structure? What are good and bad points of this data structure?
- How can you find all vertices in a face with the halfedge data structure?
- How can you find all faces that contain a vertex with the halfedge data structure?
- BONUS: Think of an algorithm to traverse every face in a manifold using this data structure.

# Lecture 10: Digital Geometry Processing

- List practical applications that you can relate to for good geometry processing algorithms.
- Give criteria for what makes a good quality mesh. Be sure to state your assumptions (e.g., good quality for what purpose?). Consider triangle size and shape and vertex out-degree.
- Give pseudocode for one iteration of Catmull-Clark subdivision
- Give pseudocode for one iteration of Loop subdivision
- Bonus: Understand how subdivision algorithms are characterized in terms of their properties (interpolation, continuity, behavior at the boundaries)
- Be prepared to calculate vertex updates in a simple example of Loop or other subdivision. (The vertex weighting masks will be given to you.)
- Understand how the  $K$  matrix of the Quadric Error error metric encodes squared distance to a plane. How can it encode the sum of squared distances to many planes? How is this idea used in generating a good error metric for mesh decimation using edge collapse?

# Lecture 10: (more) Digital Geometry Processing

- **Express distance from a plane, given a point on the plane and a normal vector**
- **Show how the  $K$  matrix (the quadric error matrix) represents squared distance from a plane**
- **Given  $K$  matrices encoding triangles in a mesh, how do we get the  $K$  matrix for each vertex?**
- **If we collapse an edge, what is the  $K$  matrix for the new vertex that is added in the edge collapse?**
- **Given a  $K$  matrix and a proposed point, what is the cost (the quadric error)?**
- **How does this cost represent distance to the original surface?**
- **Describe some techniques for improving the quality of a mesh to make it more uniform and regular.**

# Lecture 11: Geometric Queries...

- Find the closest point to a point, line, or line segment.
- Give an explicit representation for a ray.
- Give an implicit representation for a plane.
- Compute ray-triangle intersection, including checking whether the ray passed through the inside of the triangle.
- Be prepared to compute ray-primitive intersection for other primitives (e.g., a sphere) and primitive-primitive intersections (e.g., triangle-triangle or line-line)
- In all (most?) of these queries, what is the basic strategy that you use to find the closest point or perform the intersection? Think about how you use explicit and implicit representations of the geometries.

# Lecture 11: ... and Spatial Data Structures

- **Compute ray - bounding box intersection**
- **Construct a bounding box hierarchy for a given collection of objects.**
- **Calculate traversal order of a bounding box hierarchy for a given ray.**
- **What is the Surface Area Heuristic (SAH) and what goals is it trying to achieve?**
- **Explain how to choose a bounding box partition using the SAH**
- **Be able to distinguish between object-centric (primitive partitioning) acceleration structures and space-centric (space-partitioning) acceleration structures**
- **Know the difference between these acceleration structures, how to build them, how to traverse them, and when to use each type:**
  - **bounding box and bounding sphere hierarchies**
  - **KD-trees**
  - **octrees**
  - **grids**

# Lecture 13: Color

- How would you describe the emission spectrum of a light source?
- What are the rods and cones? Which are present in greater number in the human eye? What can you say about how they are distributed in the retina?
- Since color is best described by a spectrum of emissions, why is it that we can get away with just three values for color (e.g., R-G-B)?
- What are additive and subtractive models for color and when are they used?
- Describe some of the different color spaces that are used to express color.
- An alternative to RGB color space is the CIE color space, with X, Y, and Z primaries. What is Y in this color space? What problem with RGB color space does the CIE color space solve?
- BONUS: Given a color space expressed by some three-dimensional basis it be converted into any other basis through a linear operation (True or False)
- BONUS: What is gamma correction? Give an example where gamma correction is useful.

# Lecture 14: Radiometry

- **Visible light consists of a small range of wavelengths along the spectrum from gamma rays to radio waves.**
- **Energy of a photon depends on wavelength (and speed of light, and Planck's constant)**
- **What is radiant energy? ..flux? ..irradiance?**
- **Why (how) does irradiance depend on the angle between the light source and a patch of surface area?**
- **How does irradiance fall off with distance from the light source?**
- **What is a solid angle?**
- **(Bonus) What is radiance, and how many dimensions do you need to capture the radiance in a scene (i.e., to capture a light field)?**
- **What is radiant intensity?**
- **(Bonus) Characterize the spectral signatures of different familiar light sources.**
- **(Bonus) Figure out how to read a Goniometric diagram**

# Lecture 15: The Rendering Equation

- Ray tracing algorithms make use of radiance estimates to build up an image of a scene. Why is radiance the fundamental quantity of interest? How are radiance estimates used to compute the color that would be observed by a camera at a point on a material surface?
- What is the difference between radiance and irradiance? Between incident and exitant radiance?
- Explain and illustrate with a sketch all of the terms in the Rendering Equation.

# Lecture 16: The Bidirectional Reflectance Distribution Function

- **Draw diagrams to illustrate reflection for (1) a perfectly specular (mirror) surface, (2) a glossy surface, (3) a diffuse surface, (4) a retroreflective surface**
- **What is a BRDF? What are the inputs and outputs of a BRDF function?**
- **How would you measure a BRDF? If you were to mount a camera and light source on two robot arms, how many degrees of freedom (joints) would you need to do these measurements? Given a ray and a surface normal, calculate the direction of perfect reflection**
- **Given a ray, a surface normal, and indices of refraction, calculate the direction of perfect transmission using Snell's Law.**
- **What is total internal reflection? Give a detailed example of how / when it can occur.**

# Lecture 16: (more) BRDFs

- **What is Fresnel reflection? Sketch curves to illustrate the effect as we have seen in class. Label your axes. Informally, what does this effect show?**
- **What is subsurface scattering?**
- **How can we extend the idea of BRDF to subsurface scattering? What additional parameters must be sampled?**

# Lecture 17: Variance Reduction

- In rendering (global illumination), what are we integrating, i.e., what integral do we want to evaluate?
- Give a high level overview of the process of Monte Carlo integration
- Explain the terms in the Monte Carlo estimator — be familiar with both forms with and without the probability distribution function
- What is a probability density function (PDF)?
- Bonus: What is a Cumulative Distribution Function (CDF)?
- Bonus: The Inversion Method can be used to correctly draw a sample from a PDF. Sketch the overall step by step process for using the Inversion Method
- What is rejection sampling? Show how to use rejection sampling to sample area of a circle, volume of a sphere, directions on a sphere, and solid angles from a hemisphere.

# Lecture 17: (more) Variance Reduction

- **What is Expected Value? .. Variance? .. Bias?**
- **What is Importance Sampling? How is it used in cosine-weighted sampling of the hemisphere?**
- **What is the Monte Carlo method (in general)?**
- **Explain how a Monte Carlo method can be used to solve the Rendering Equation.**
- **What is Russian Roulette and why do we need it?**
- **How can we use Russian Roulette and still have an unbiased estimator?**

# Lecture 17: (and more) Variance Reduction

- Be familiar with the following expression for Monte Carlo integration. What is the role of each term?

$$I = \lim_{n \rightarrow \infty} V(\Omega) \frac{1}{n} \sum_{i=1}^n f(X_i)$$

- Give an example of how we can reduce variance in our rendered results in a path tracing algorithm without increasing the number of samples.
- What does it mean for an estimator to be consistent?
- What does it mean for an estimator to be unbiased?
- Give a concrete example of how a renderer could give a biased estimate of an image. Is the renderer in your example consistent? Explain your answer.
- Give five examples of how you can reweight samples in a pathtracing algorithm in order to do importance sampling.
- What are the main ideas behind bidirectional path tracing?
- (Bonus) How would you enumerate all possible paths in a scene?
- How does Metropolis-Hastings sampling work?
- Assume you have code to generate random paths and code to mutate existing paths. Write pseudocode for Metropolis-Hastings path tracing.

# Lecture 17: (and even more) Variance Reduction

- **What is Stratified Sampling?**
- **Why is it preferred to random sampling?**
- **Hammersley and Halton points are pseudo random sampling techniques to generate points with low discrepancy. What is discrepancy? Why do we want to generate low discrepancy samples?**
- **Give a concise one sentence description of each of the following rendering algorithms that makes it clear the differences between them. Use a diagram to illustrate your description:**
  - **Rasterization**
  - **Ray casting**
  - **Ray tracing**
  - **Path tracing**
  - **Bidirectional path tracing**
  - **Metropolis Light Transport**
  - **Photon Mapping**
  - **Radiosity**
- **Which of these algorithms are best for capturing reflective surfaces? caustics? color bleeding? subsurface scattering? refraction? ...**

# Lecture 18: Intro to Animation

- **How were the first animations created?**
- **What is rotoscoping and how is it used?**
- **Describe some pros and cons of hand animation, 3D computer animation, simulation, and motion capture**
- **What are some of the principles of animation?**

# Lecture 19: Splines

- **What is a Hermite spline?**
- **Give properties in terms of continuity, interpolation, and local control**
- **Derive a Hermite spline from its constraints**
- **Express it in matrix form**
- **What type of spline has  $C^2$  continuity and interpolation, but not local control?**
- **What type of spline has  $C^2$  continuity and local control, but does not interpolate its key points?**
- **Bezier, Hermite, and Catmull-Rom splines are really all the same thing. Any one representation can be converted to any of the others. Explain the differences between them.**

# Lecture 19: (more) Splines

- Be able to express a Hermite spline in different ways — as a cubic polynomial, in matrix form, or derive it from its control parameters.
- How do we ensure continuity between cubic spline segments?  $C_0$  continuity? ..  $C_1$  continuity? .. is  $C_2$  continuity possible in general?

# Lecture 20: Kinematics

- Describe the difference between forward and inverse kinematics
- When would you want to use each? Give examples.
- Solve for the position of the end effector of a simple robot using forward kinematics
- What is the Jacobian?
- Compute the Jacobian of a simple robot to express how its end effector moves in relation to its joint motion
- How do we use the Jacobian to do inverse kinematics? (It is ok to describe the gradient based approach based on the Jacobian transpose which we used in class)

# Lecture 21: Simulation and ODEs

- What is the “animation equation”?
- What is the difference between an ODE and a PDE? Give some examples of systems we can simulate by integrating an ODE.
- Sketch an overall system for simulating an ODE using a block diagram. Be clear about what is the state, how you advance the state forward in time, and what integrator you are choosing.
- When is the Euler-Lagrange equation useful?
- Be able to work through a simple example of obtaining dynamic equations of motion using Lagrangian mechanics.
- Describe how to put together a mass-spring system to simulate cloth.
- What are the forces on each cloth “particle”?
- What is Forward Euler integration and what is its disadvantage? Can you show a simple example where it fails?
- What is Backward Euler integration and what are its pros and cons?
- What is Symplectic Euler integration?

# Lecture 22: Physically Based Animation and PDEs

- What is the difference between a PDE and an ODE? Give examples of when you might use each one and why.
- Interpret this sentence using an equation and a diagram: Solving a PDE looks like *“use neighbor information to get velocity (...and then add a little velocity each time)”*
- Burger’s equation is first order in time and second order in space. What does that mean? What are the orders of the Laplace equation? The heat equation? The wave equation? Be able to figure out the order of an equation from an expression of the equation itself.
- What are examples of questions we can answer using the Laplace equation, the heat equation, and the wave equation respectively?
- Outline the basic strategy for solving a PDE.
- What is the Laplace operator? Write it out as a sum of partial derivatives.
- Copy down the heat equation from the slides. Write out the process of solving this equation using Forward Euler integration.
- Copy down the wave equation from the slides. Write out the process of solving this equation using Forward Euler integration.

# Lecture 22: (more) Physically Based Animation and PDEs

- Be able to use the grid version of the Laplacian to do smoothing on a grid.

# Lecture 23: Optimization

- Describe some problems in Computer Graphics where optimization is important.
- Be able to describe an optimization problem in standard form and give a couple of simple examples.
- How do you know you have a minimum of an objective function in an optimization problem without constraints? What properties must be true?
- What is meant by convexity of the domain in an optimization problem? convexity of the objective? Give examples of each. Why do we care about convexity in optimization?
- Describe some pros and cons of gradient descent approaches to optimization
- How can we improve upon gradient descent?