

A2.5: MeshEdit

Global Ops Edition

“I hate meshes. I cannot believe how hard this is.

Geometry is hard.”

— David Baraff, Senior Research Scientist, Pixar Animation Studios

Halfedge Data Structure

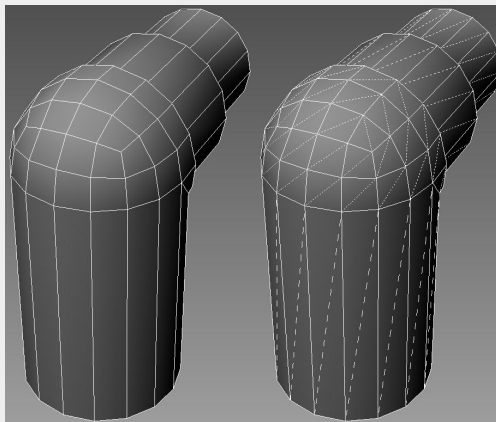
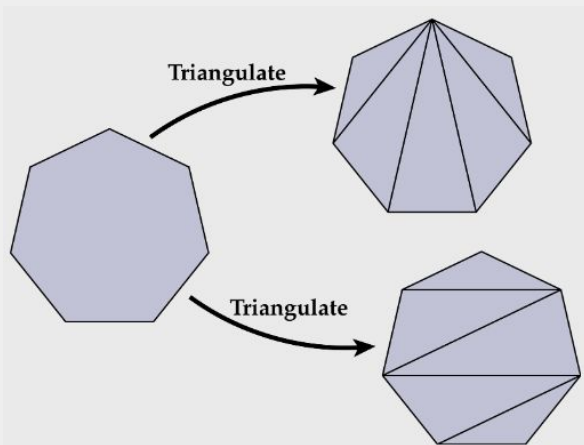
```
class Halfedge {  
public:  
    //connectivity:  
    HalfedgeRef twin; //halfedge on the other side of edge  
    HalfedgeRef next; //next halfedge ccw around face  
    VertexRef vertex; //vertex this halfedge is leaving  
    EdgeRef edge; //edge this halfedge is half of  
    FaceRef face; //face this halfedge borders
```

- Triangulation
- Linear Subdivision
- Catmull-Clark Subdivision
- Loop Subdivision
- Isotropic Remeshing
- Simplification

- **Triangulation**
- Linear Subdivision
- Catmull-Clark Subdivision
- Loop Subdivision
- Isotropic Remeshing
- Simplification

Triangulation

- Goal: split each non-triangular face in mesh into triangles
 - No strict way to triangulate the faces, multiple patterns are acceptable

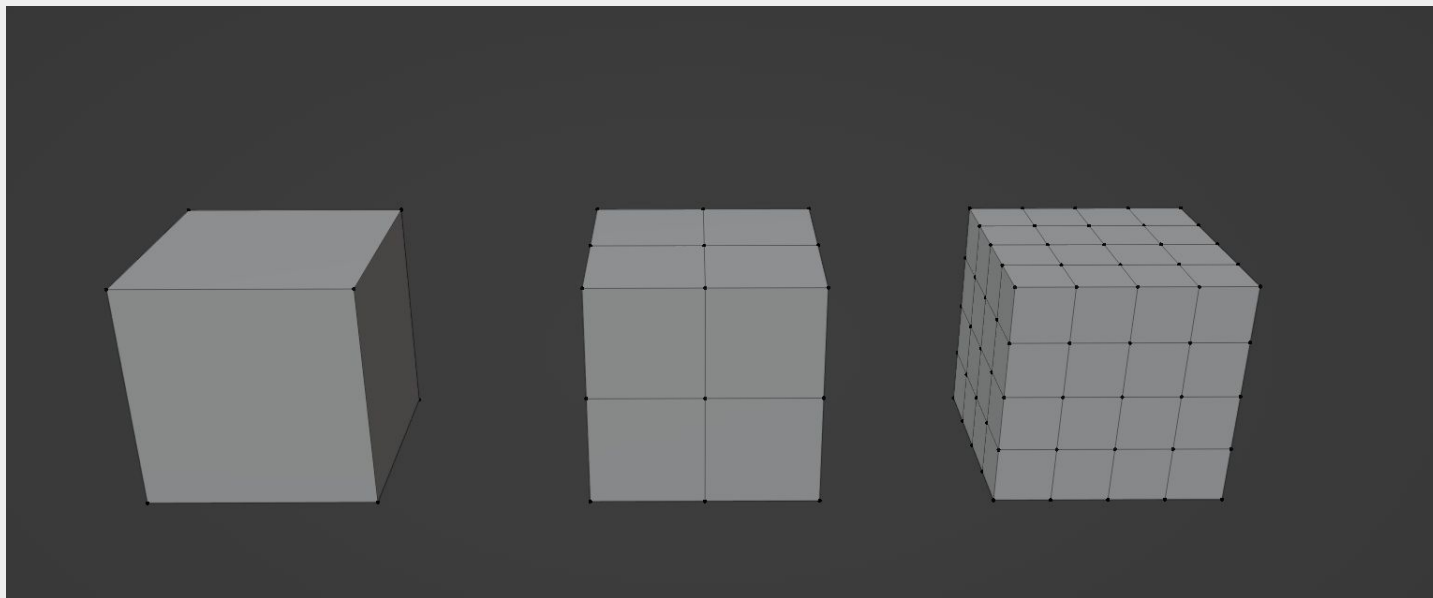


- Most 3D engines (Blender, Unity, Scotty 3D, etc) automatically triangulate meshes before rendering as rasterizing triangles is simpler than rasterizing polygons of higher order
- Triangulated meshes can be used as inputs for other geometric processing algorithms that require tri meshes, like isotropic remeshing

- Triangulation
- **Linear Subdivision**
- Catmull-Clark Subdivision
- Loop Subdivision
- Isotropic Remeshing
- Simplification

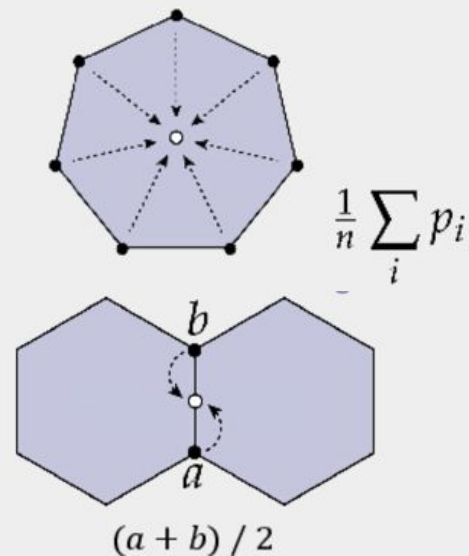
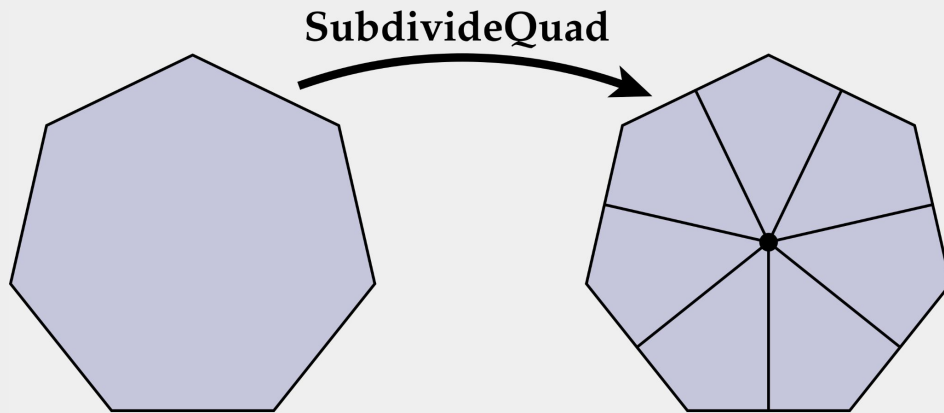
Linear Subdivision

-Goal: Increase fidelity of mesh by splitting existing geometry into quads



Linear Subdivision

- Split each n-gon into quads

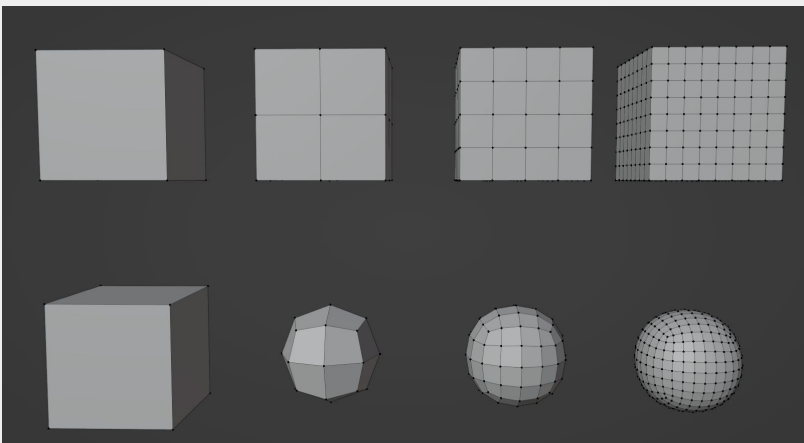


- New central vertex position is average of vertex positions on the perimeter
- New vertex positions on the edges are on the midpoint of the pre-existing edge

Linear Subdivision

- In contrast to Catmull-Clark, linear subdivision does not modify the vertex positions of the original mesh, resulting in no smoothing

Linear ->

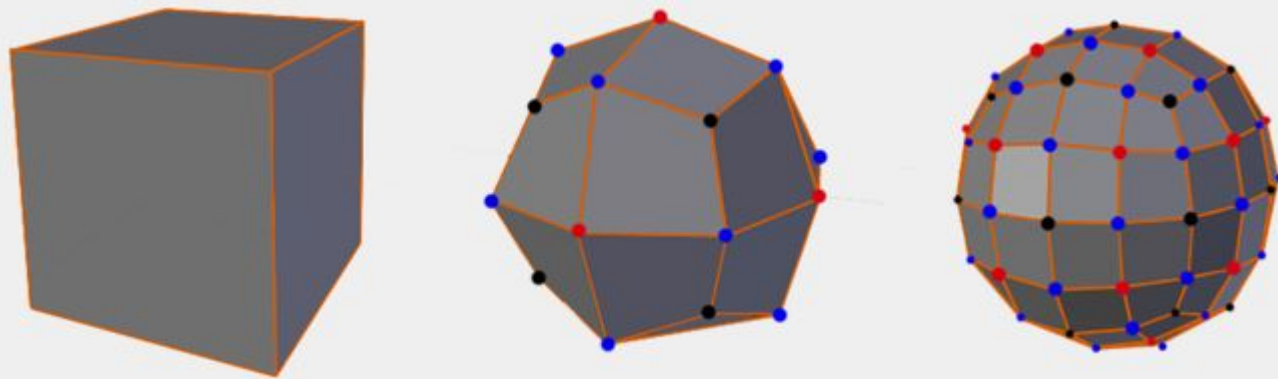


- Useful for when you want to increase fidelity of mesh but preserve its silhouette
 - eg to preprocess mesh before using displacement mapping or procedurally modifying vertex positions with vertex shaders

- Triangulation
- Linear Subdivision
- **Catmull-Clark Subdivision**
- Loop Subdivision
- Isotropic Remeshing
- Simplification

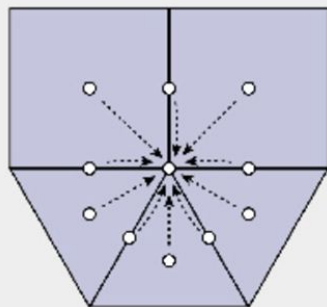
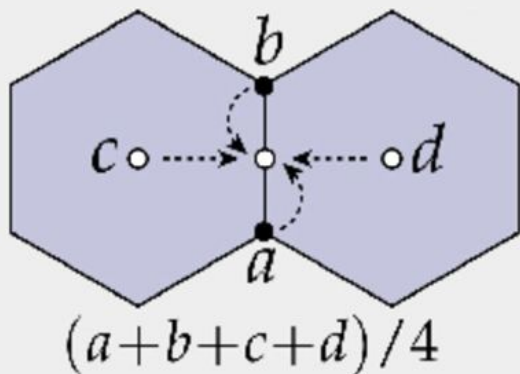
Catmull-Clark Subdivision

-Goal: Increase fidelity of mesh by splitting existing geometry into quads and smoothing the resulting geometry



Catmull-Clark Subdivision

- Split step is same as linear subdivision
- New edge and vertex coordinates require a bit more math



$$\frac{Q + 2R + (n - 3)S}{n}$$

Vertex Coords

n - vertex degree

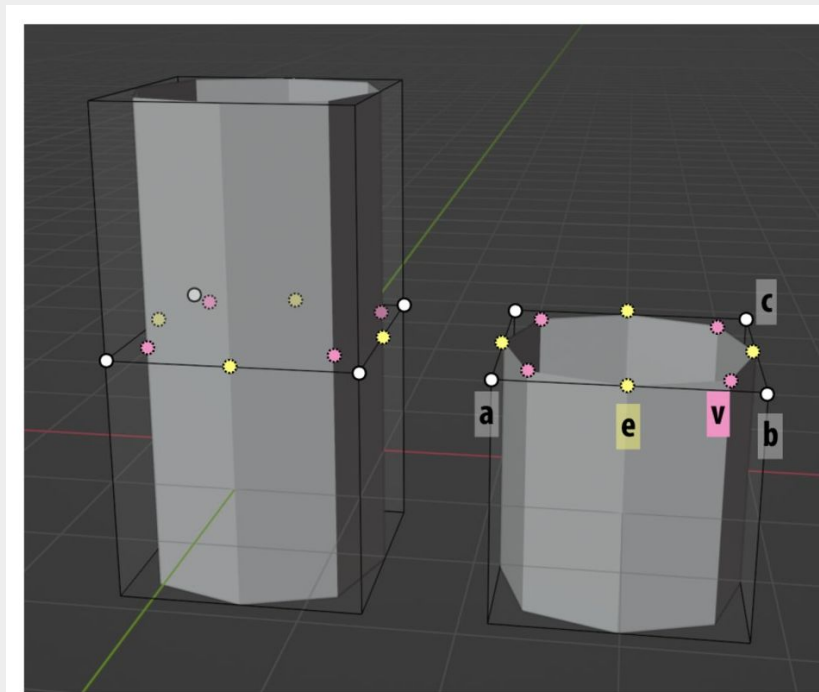
Q - average of face coords around vertex

R - average of edge coords around vertex

S - original vertex position

Catmull-Clark Subdivision

- Boundary edges



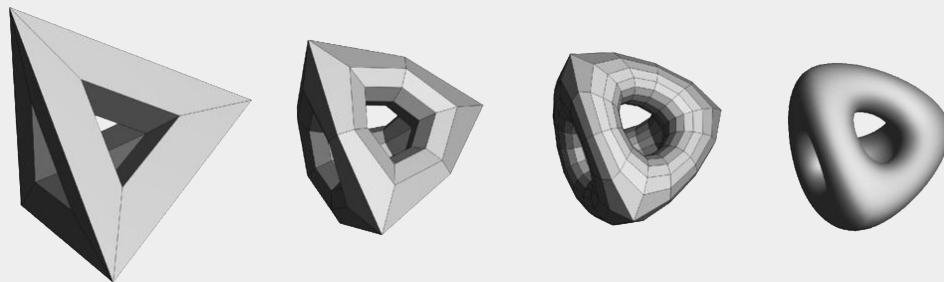
Idea: want the subdivision to “behave the same” if tube gets cut in half

=> edge points: midpoints
 $e = (a + b) / 2$

=> vertex positions:
 $v = 1/8a + 1/8c + 3/4 b$

Catmull-Clark Subdivision

- Smooths mesh in a appealing way, removes sharpness of low-poly mesh
 - Most effective for more naturalistic meshes, like human characters
 - Nothing in real life is completely sharp – everything has a slight bevel to it
- Oftentimes 3D programs automatically apply multiple Catmull-Clark subdivisions right before rendering
 - So meshes remain computationally cheap and easily malleable when editing, but smooth and high-resolution in the final render



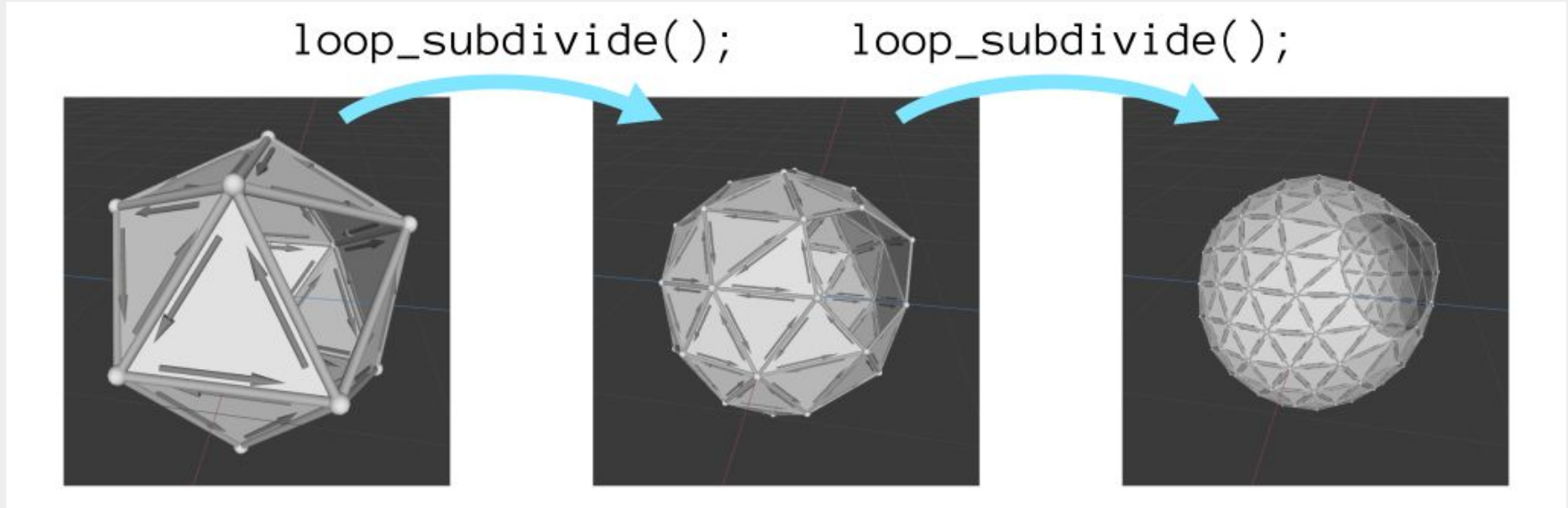
- Triangulation
- Linear Subdivision
- Catmull-Clark Subdivision
- **Loop Subdivision**
- Isotropic Remeshing
- Simplification

Loop Subdivision

- Catmull-Clark subdivision works quite well, but is there a better subdivision scheme we can use for triangulated meshes?
 - Yes! We can use **loop subdivision**
- We know that all the primitives in a mesh are triangles, so we can just split them into smaller triangles to subdivide the mesh
- How do we achieve this?
 - Split each triangle into 4 smaller triangles
 - Assign coordinates for new vertices
 - Assign coordinates for old vertices

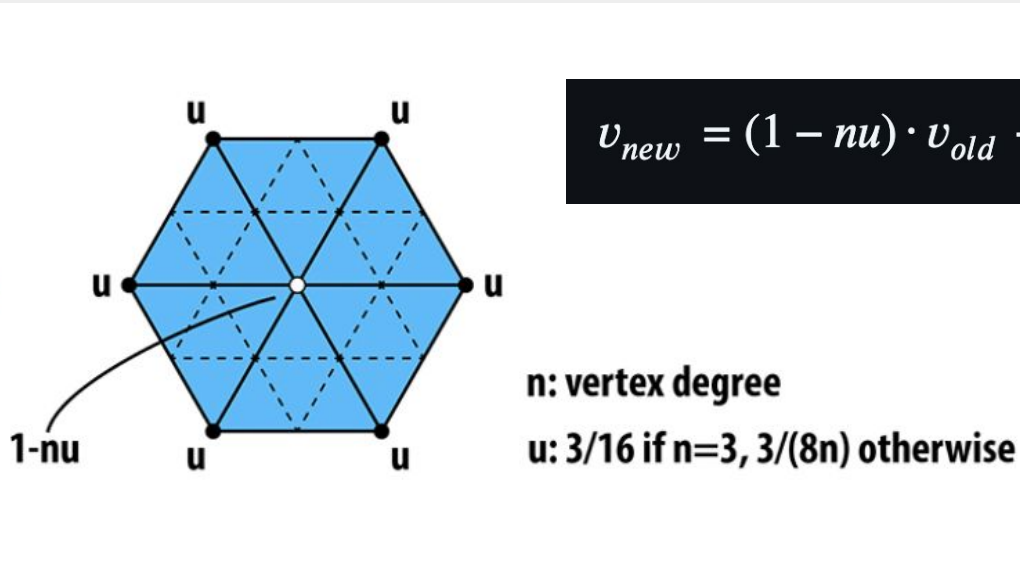
Loop Subdivision

- What does this look like when we actually apply it to a mesh?



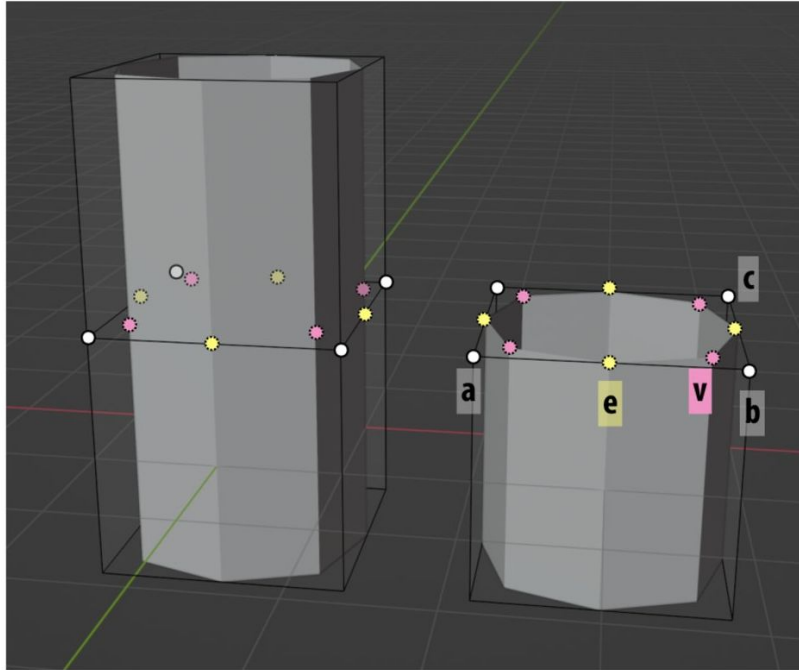
Loop Subdivision

- Compute the new positions for every existing vertex in the mesh using the loop subdivision rule
 - Be sure to treat **boundary and interior vertices** appropriately!



Loop Subdivision

- Boundary vertices



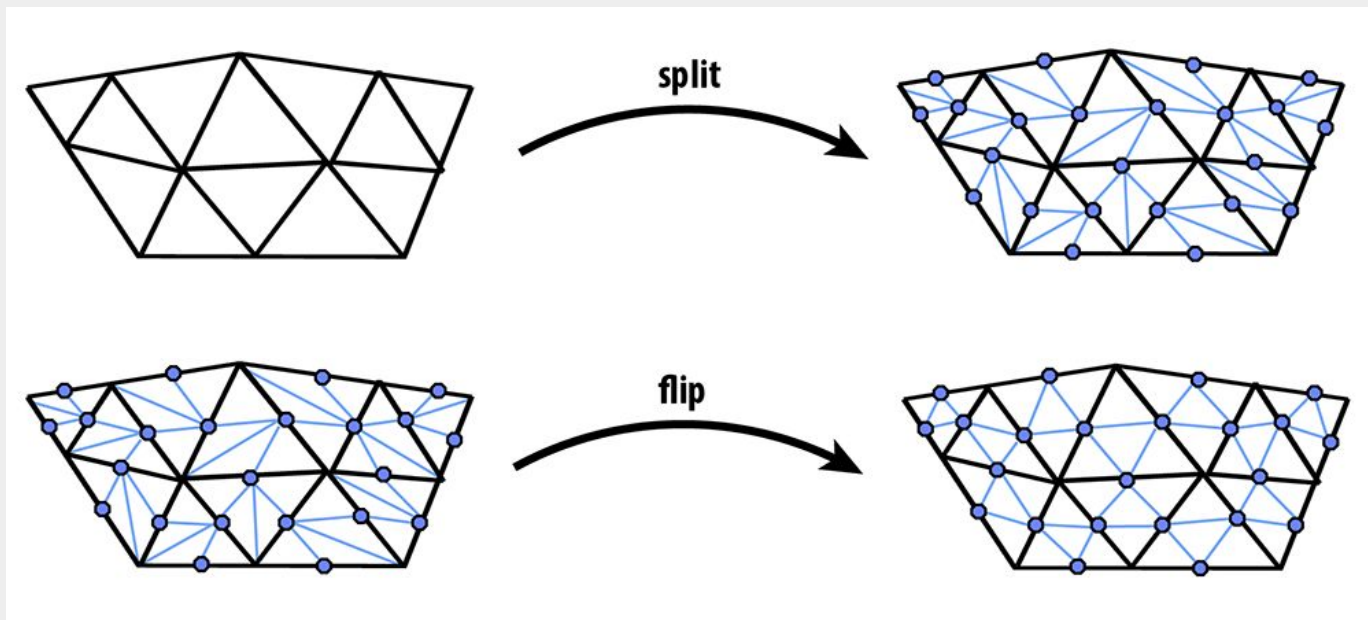
Idea: want the subdivision to “behave the same” if tube gets cut in half

=> edge points: midpoints
 $e = (a + b) / 2$

=> vertex positions:
 $v = 1/8a + 1/8c + 3/4 b$

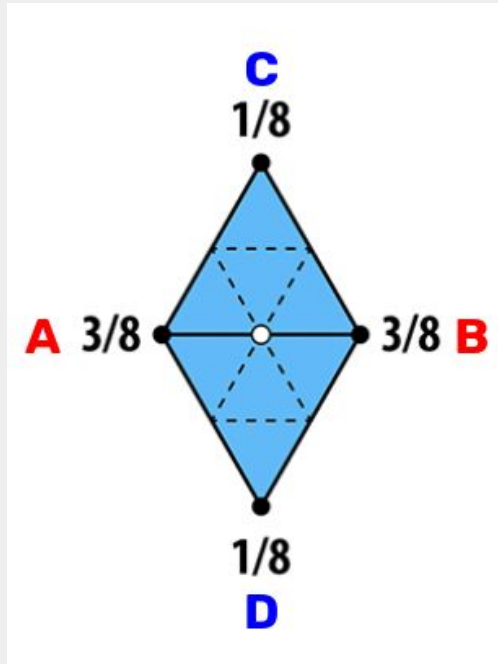
Loop Subdivision

- Split each triangle into 4 smaller triangles
- We can do this using the local ops that we implemented in A2.0
 - Important to keep track of old edges and new edges!



Loop Subdivision

- Compute the new positions for every **new** vertex in the mesh using the loop subdivision rule

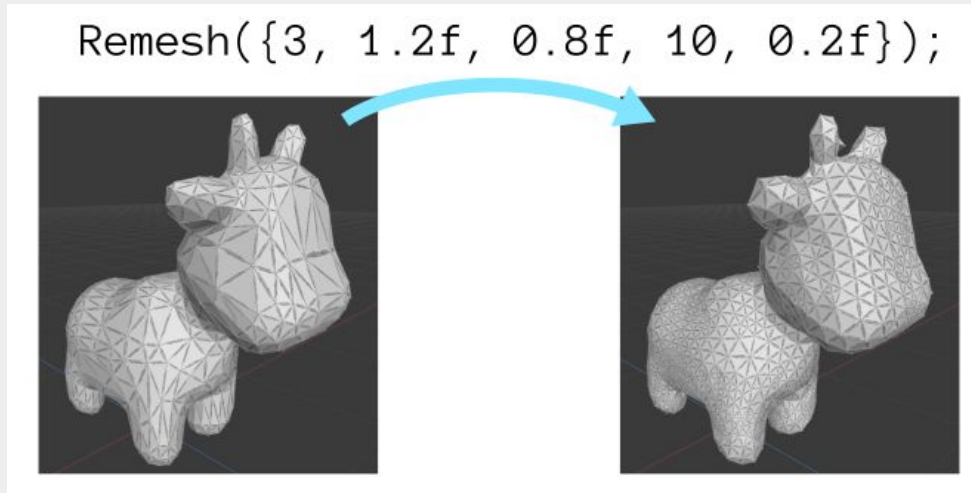


$$v_{new} = \frac{3}{8} \cdot (A + B) + \frac{1}{8} \cdot (C + D)$$

- Triangulation
- Linear Subdivision
- Catmull-Clark Subdivision
- Loop Subdivision
- **Isotropic Remeshing**
- Simplification

Isotropic Remeshing

- Goal: Produce a mesh with a nicer edge length and angle distribution by repeatedly **splitting edges** that are too long, **collapsing edges** that are too short, and applying **smoothing** (tangent to the surface normal) to vertex positions.



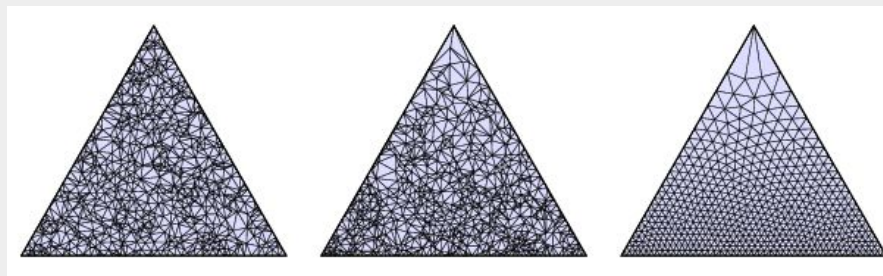
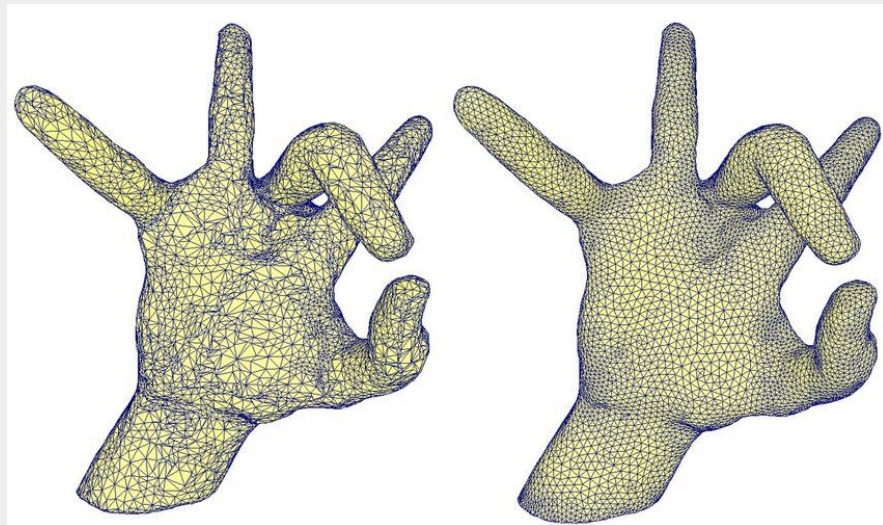
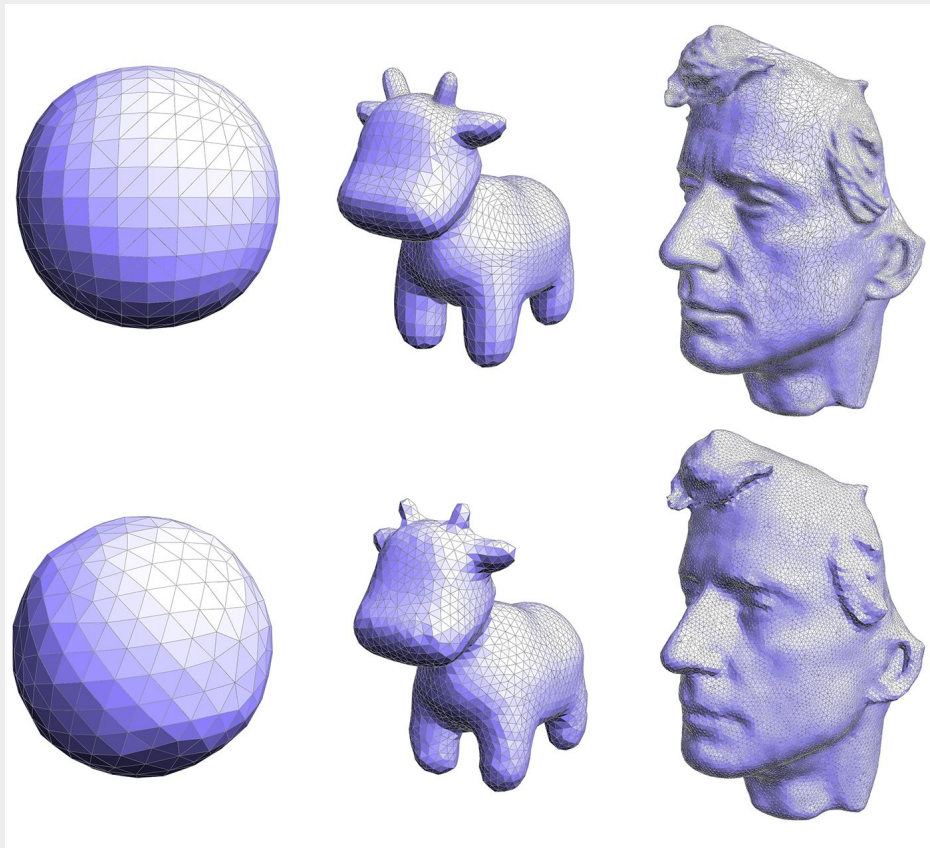
Isotropic Remeshing

- Trying to make mesh more “uniform”
 - i.e. more equilateral triangles, vertex degrees closer to 6, angles close to 60 deg
- How do we achieve this?
 - If an edge is too long, split it.
 - If an edge is too short, collapse it.
 - If flipping an edge improves the degree of neighboring vertices, flip it.
 - Move vertices toward the average of their neighbors.

Isotropic Remeshing

- If an edge is too long, split it. If an edge is too short, collapse it.
 - L = mean edge length
 - Want edges to be $4L/5 < e < 4L/3$
 - See `Isotropic_Remesh_Parameters` Edge Mesh/Isotropic_Remesh_Parameters
- If flipping an edge improves the degree of neighboring vertices, flip it.
 - Anytime it reduces the total deviation from regular degree (6)
 - Note flipping an edge might destroy it!
- Move vertices toward the average of their neighbors.
 - See `Vertex::neighborhood_center()`
 - First compute new positions, THEN reassign
 - Want to move gently towards center

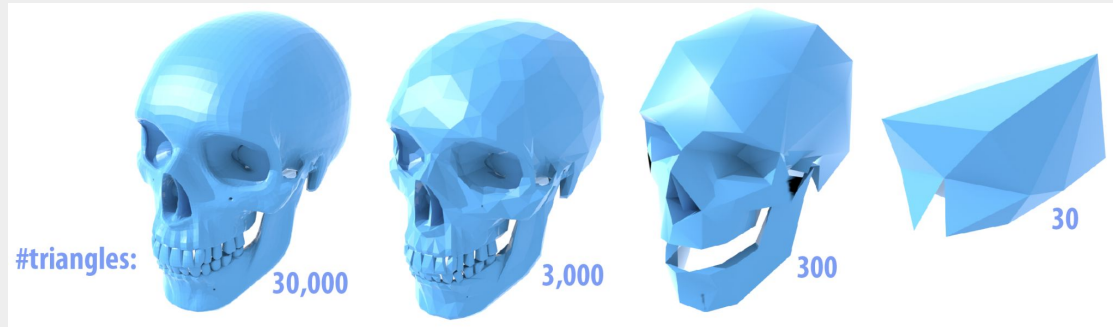
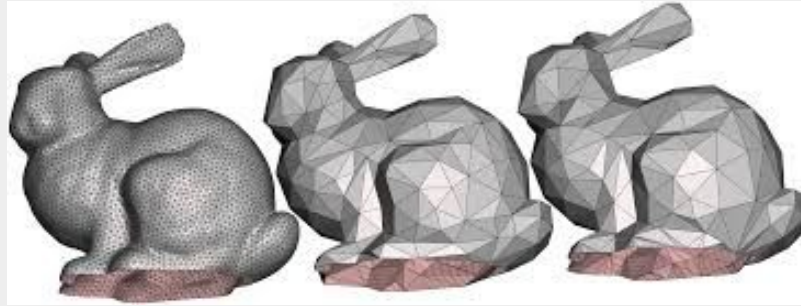
Isotropic Remeshing



- Triangulation
- Linear Subdivision
- Catmull-Clark Subdivision
- Loop Subdivision
- Isotropic Remeshing
- **Simplification**

Simplification

- Goal: Reduce the amount of triangles our mesh has while retaining its overall shape



Quadric Error Simplification

- Developed at CMU!
- Iteratively **collapse** edges until we reach the desired number of triangles
- But which edges do we use?

High Level:

- Let's find edges that, when collapsed, approximate the original mesh as best as possible
- We do this with quadric error matrices!

Quadric Error Simplification

- Quadric Error Matrices:
 - Represent the distance (“error”) from one of the original vertices of the mesh to the edge we are considering collapsing
- We can then take the derivative of this error, set it equal to 0, and find the best vertex position along this collapsed edge that most accurately approximates the original mesh
- Repeat this until we collapse the desired number of edges
- The writeup goes through all the exact math and gives a good guide on how to structure your implementation

Simplification

