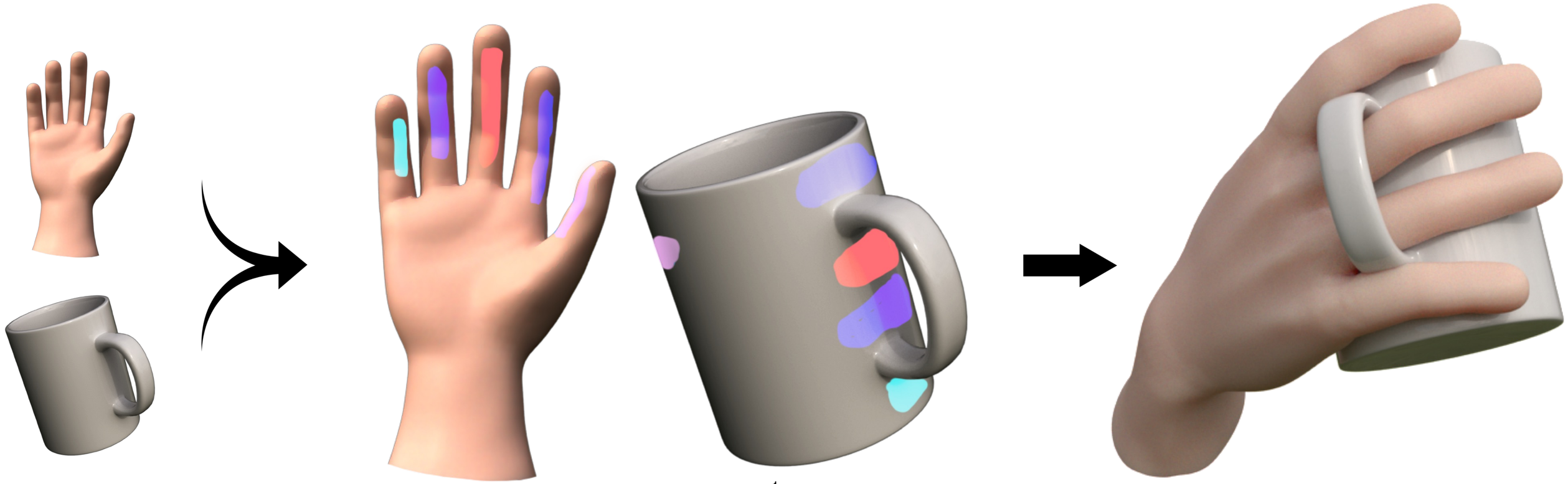
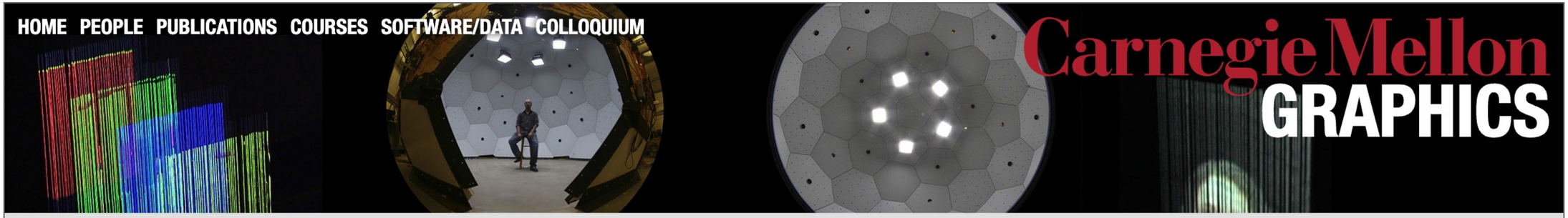
Contact Areas for Dexterous Grasping, Manipulation, and Beyond

Arjun Lakshmipathy

February 26, 2025



Shameless Plug for CMU Graphics Lab



People

FACULTY


Alexandra Ion


Ioannis Gkioulekas


Jessica Hodgins

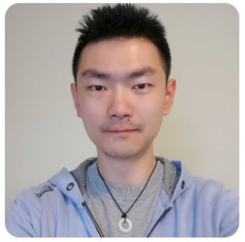

Jim McCann


Jun-Yan Zhu


Keenan Crane


Levent Burak Kara


Matthew O'Toole

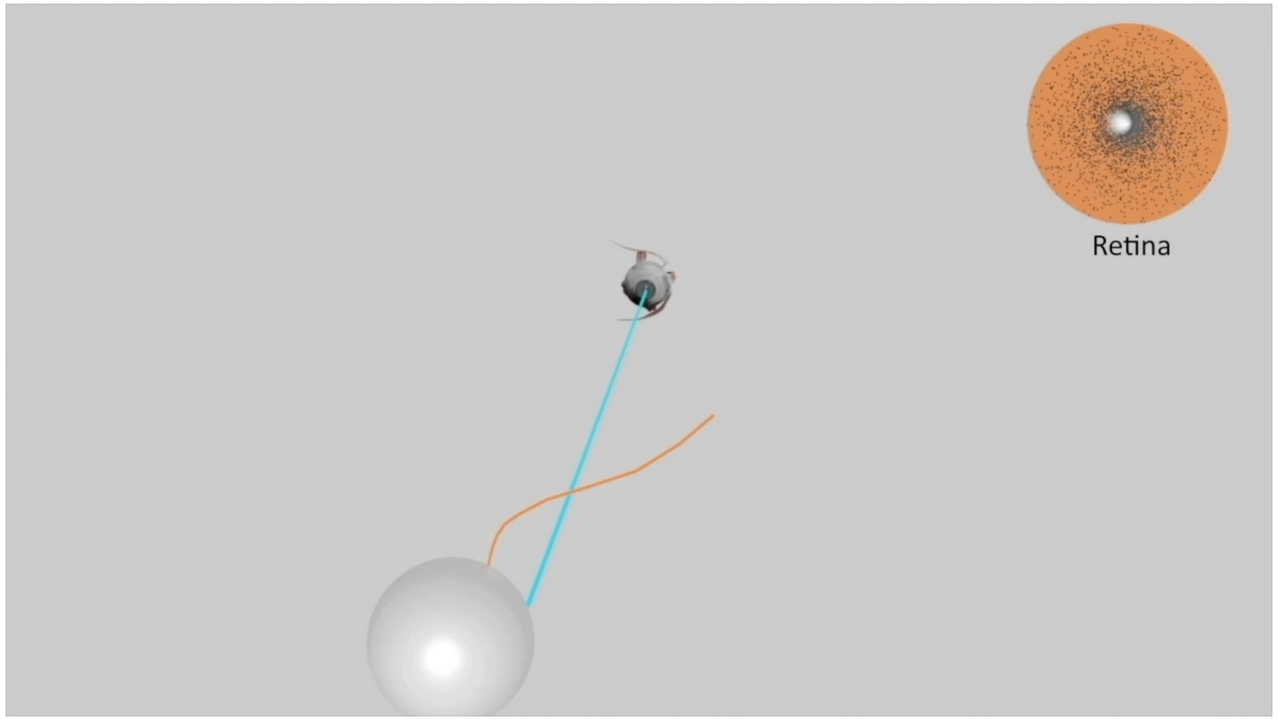

Minchen Li


Nancy Pollard


Srinivasa Narasimhan

About Me

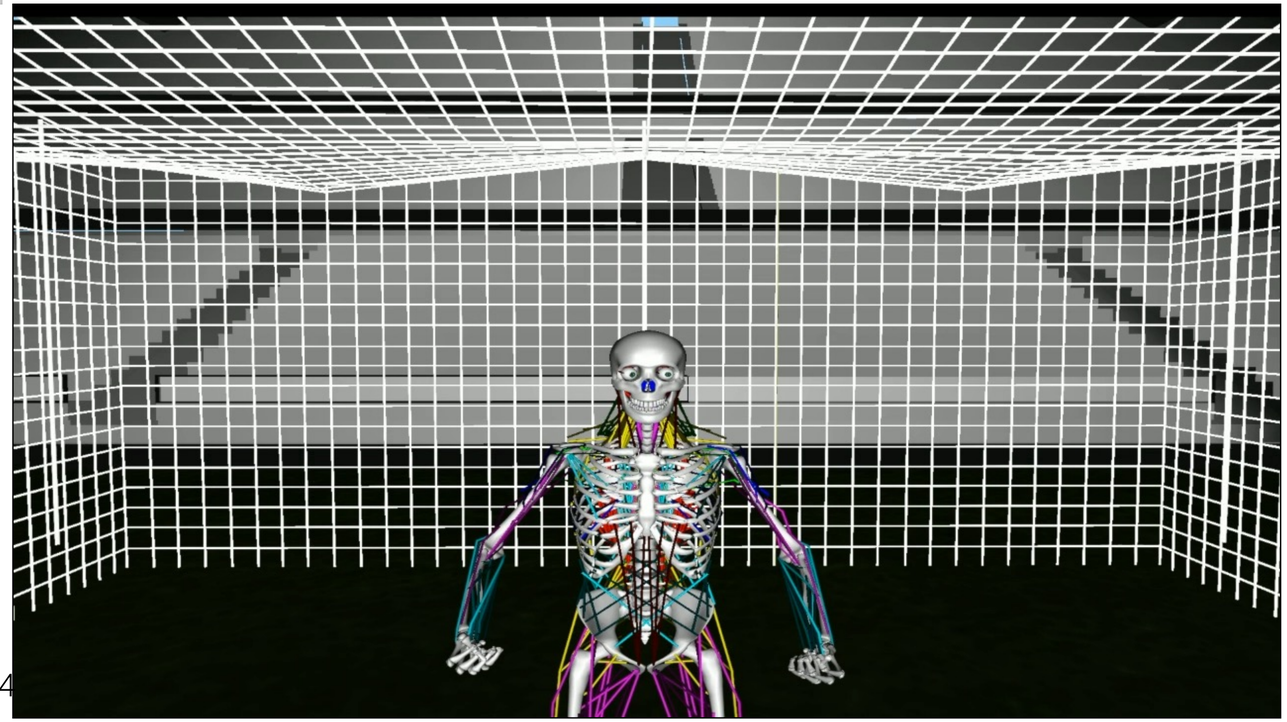
- 5th year PhD student, advised by Nancy
- Interested in **Elegant and Effective Representations for Embodied Agents**
- Background: Graphics (Animation), Robotics
- Started my research journey at UCLA as a Masters Student

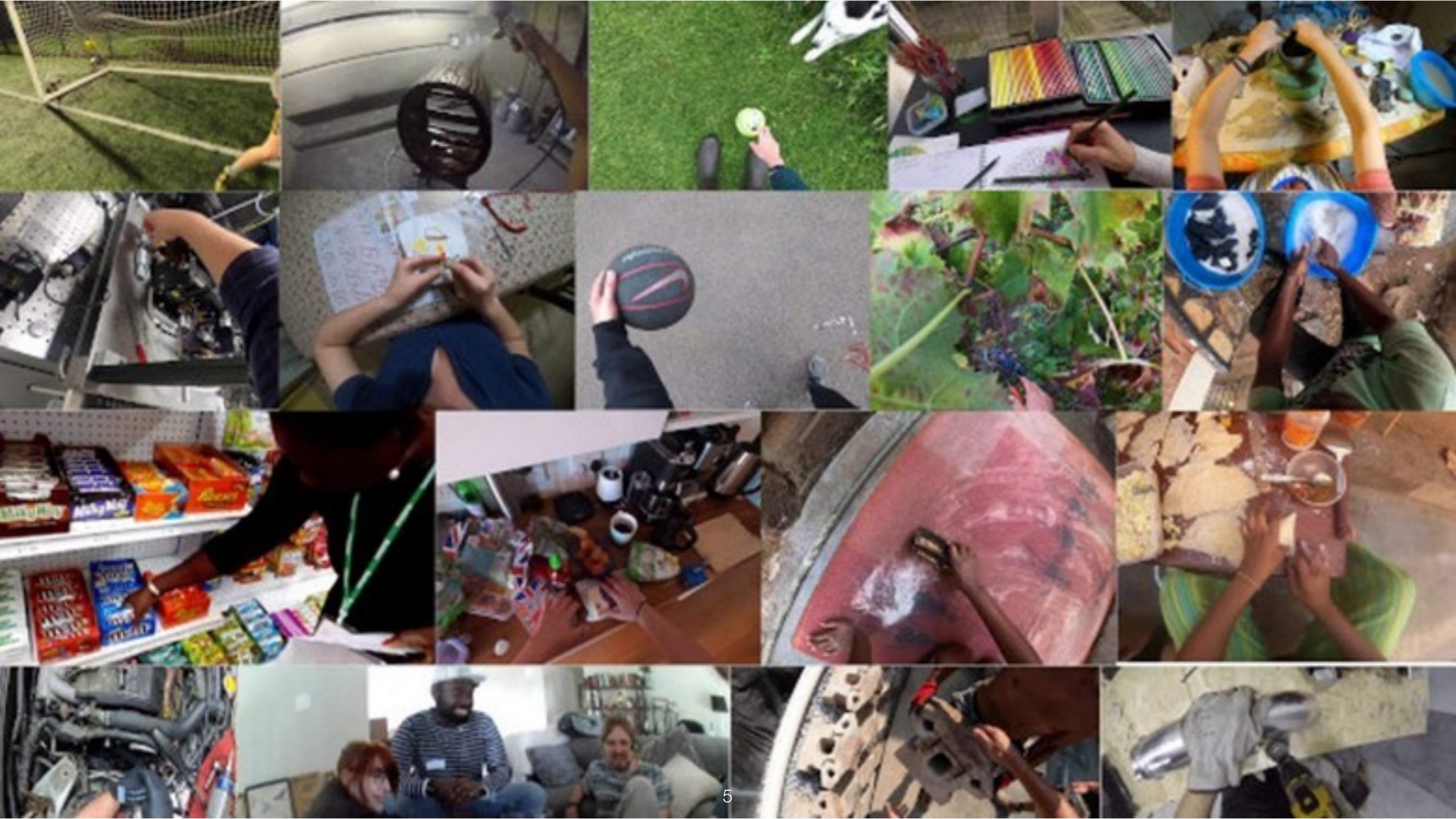


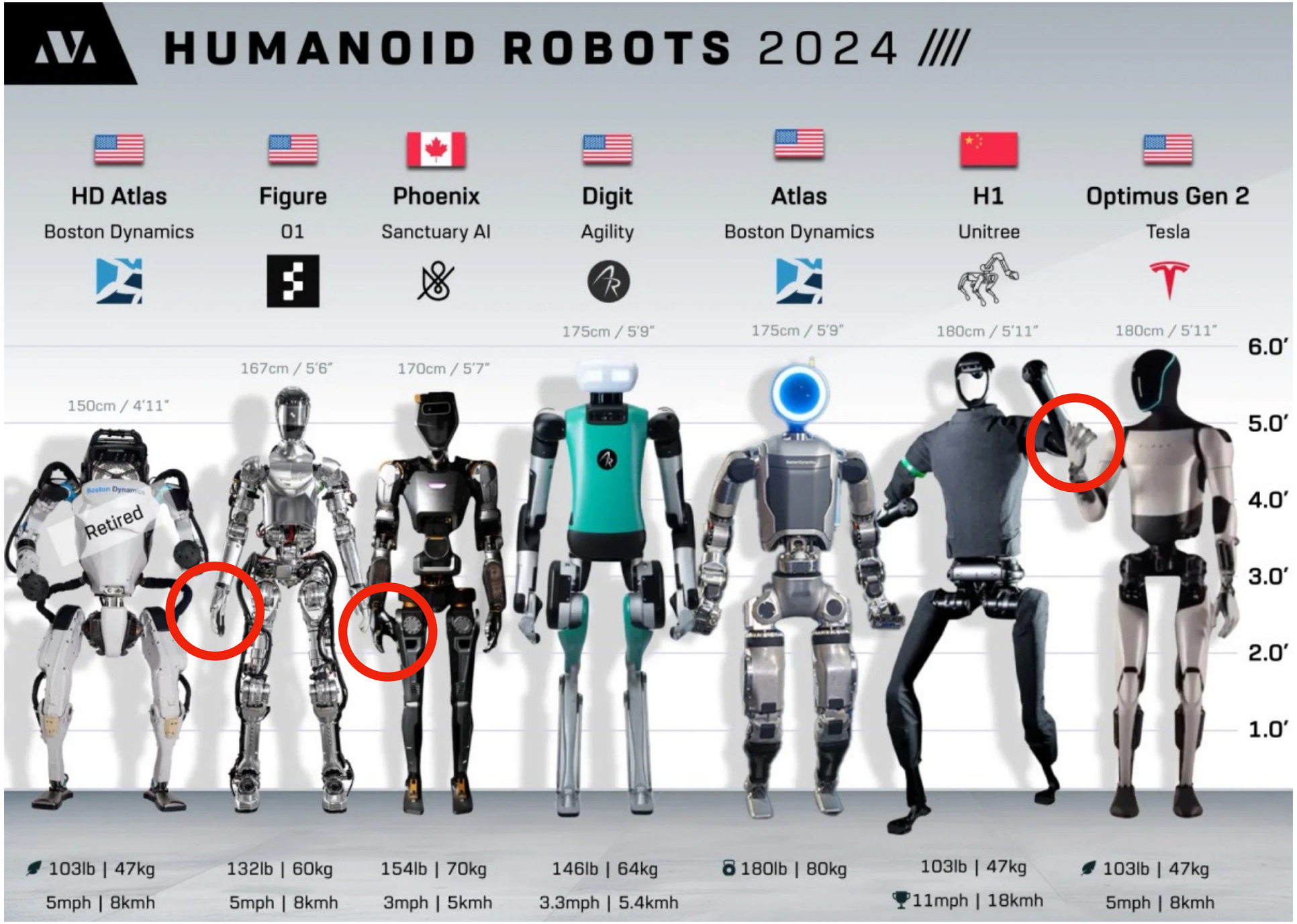
Biomimetic Eye Modeling & Deep Neuromuscular Oculomotor Control
[SIGGRAPH Asia 2019]

NeuralX

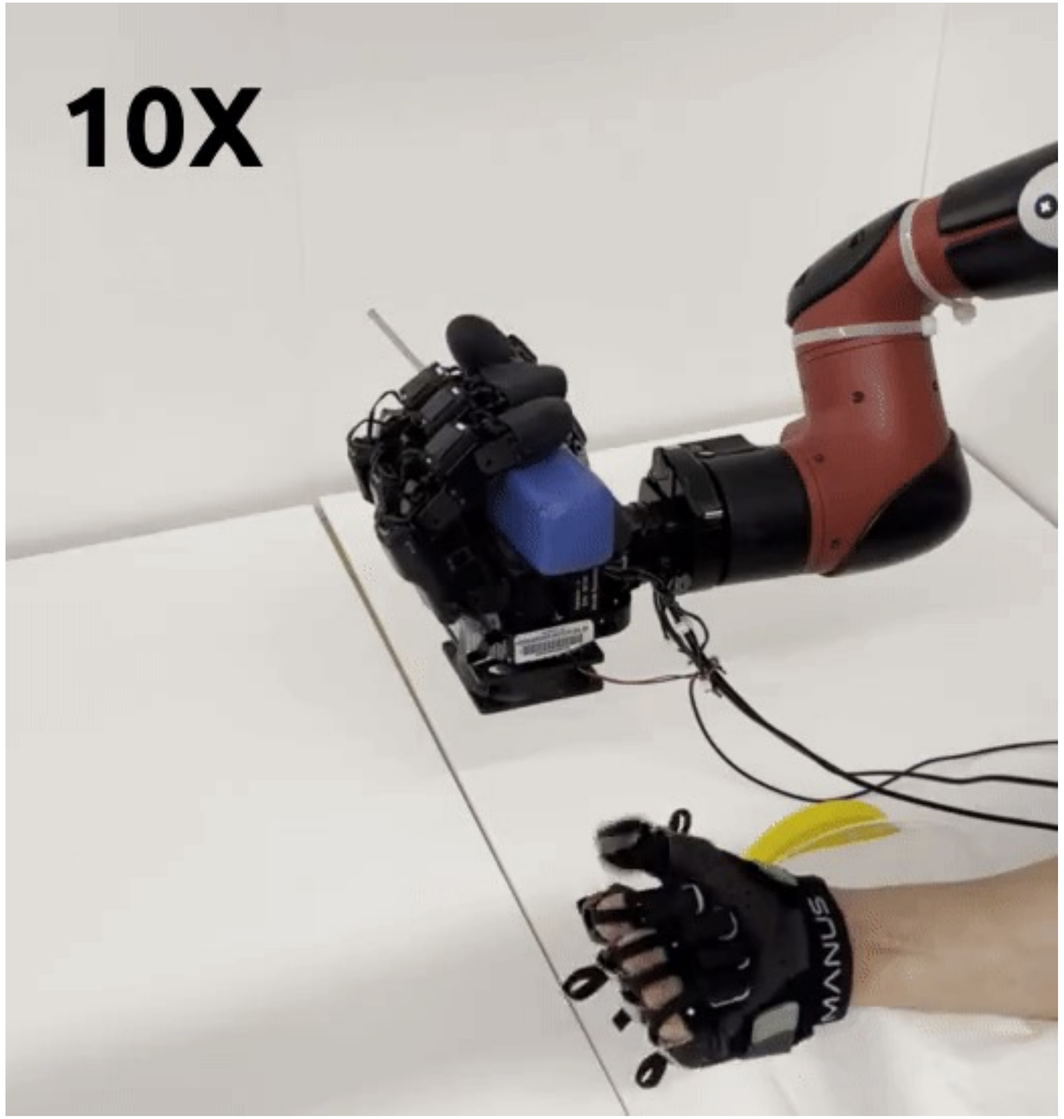
Deep Learning of Neuromuscular and Visuomotor Control in a Biomimetic Simulated Humanoid
[RA-L 2020]



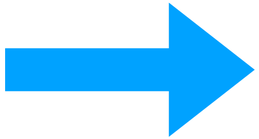



HUMANOID ROBOTS 2024
HD Atlas
Boston Dynamics
Figure
01
Phoenix
Sanctuary AI
Digit
Agility
Atlas
Boston Dynamics
H1
Unitree
Optimus Gen 2
Tesla
150cm / 4'11"
167cm / 5'6"
170cm / 5'7"
175cm / 5'9"
175cm / 5'9"
180cm / 5'11"
180cm / 5'11"
Retired
6.0'
5.0'
4.0'
3.0'
2.0'
1.0'
103lb | 47kg
5mph | 8kmh
132lb | 60kg
5mph | 8kmh
154lb | 70kg
3mph | 5kmh
146lb | 64kg
3.3mph | 5.4kmh
180lb | 80kg
103lb | 47kg
11mph | 18kmh
103lb | 47kg
5mph | 8kmh

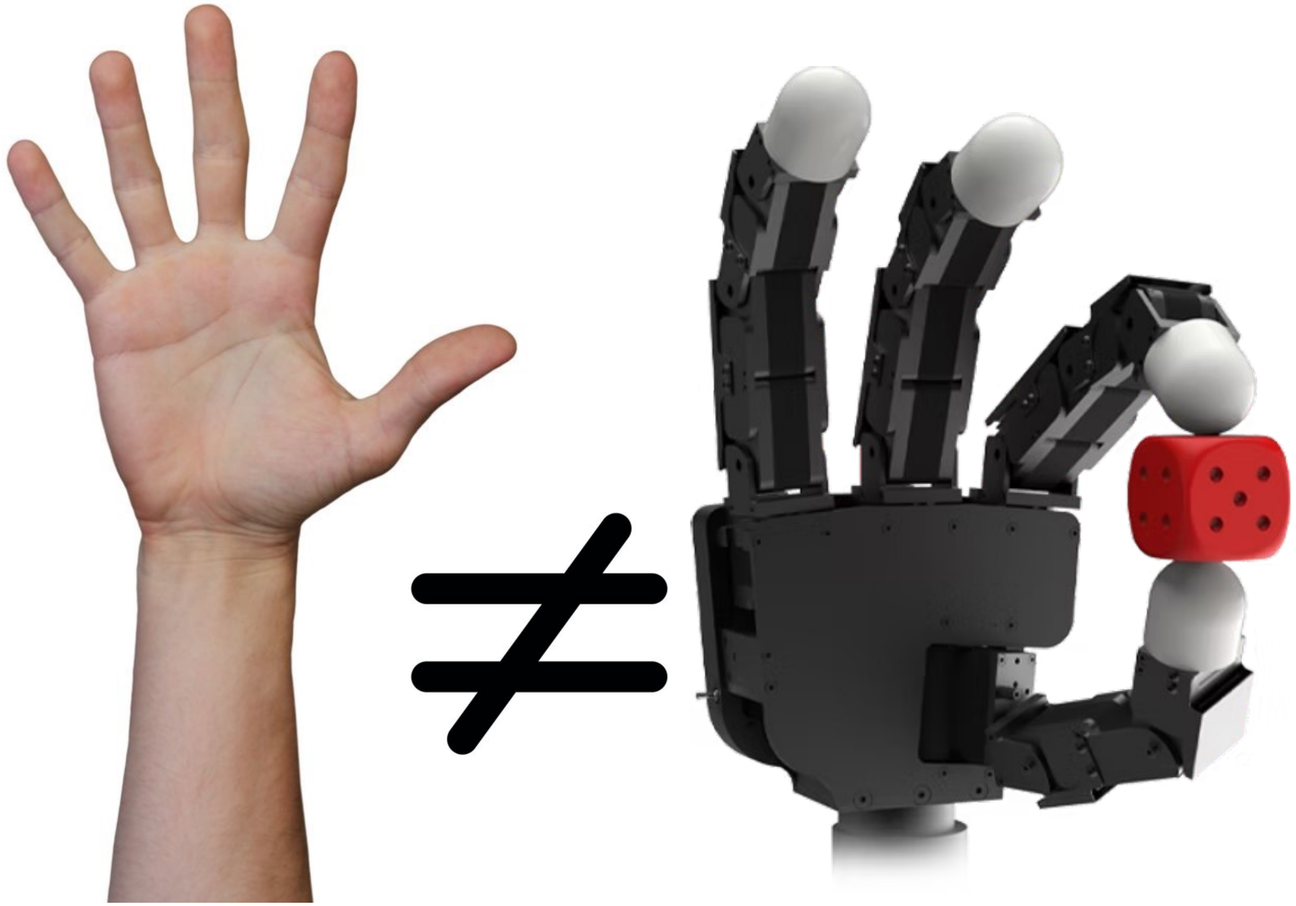
The Current State of the Art



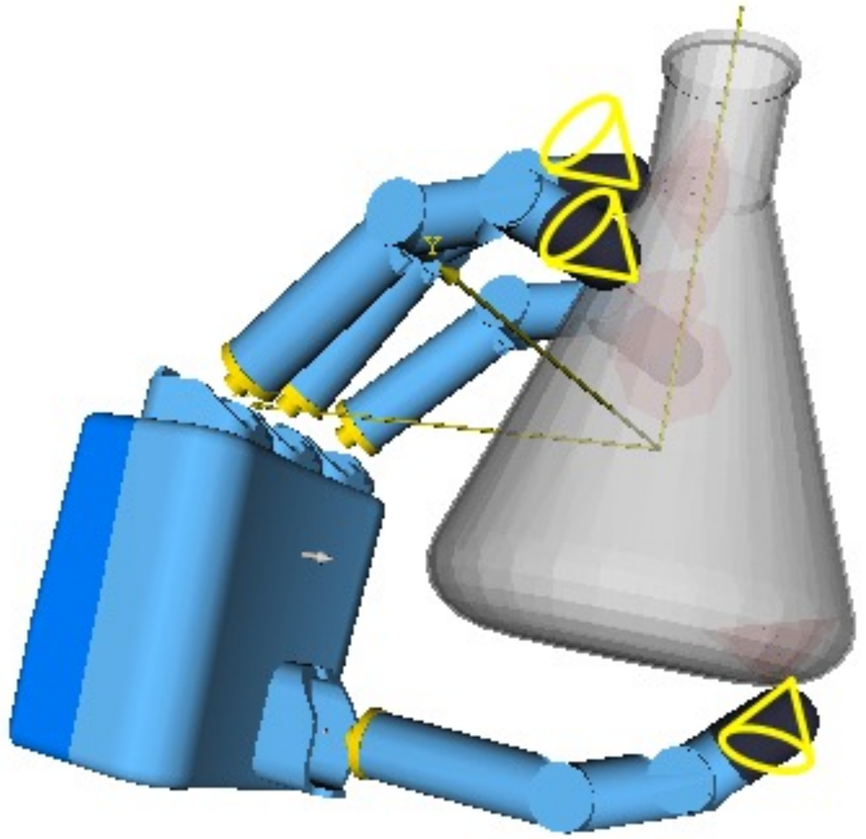
Black Box ML



The Problems

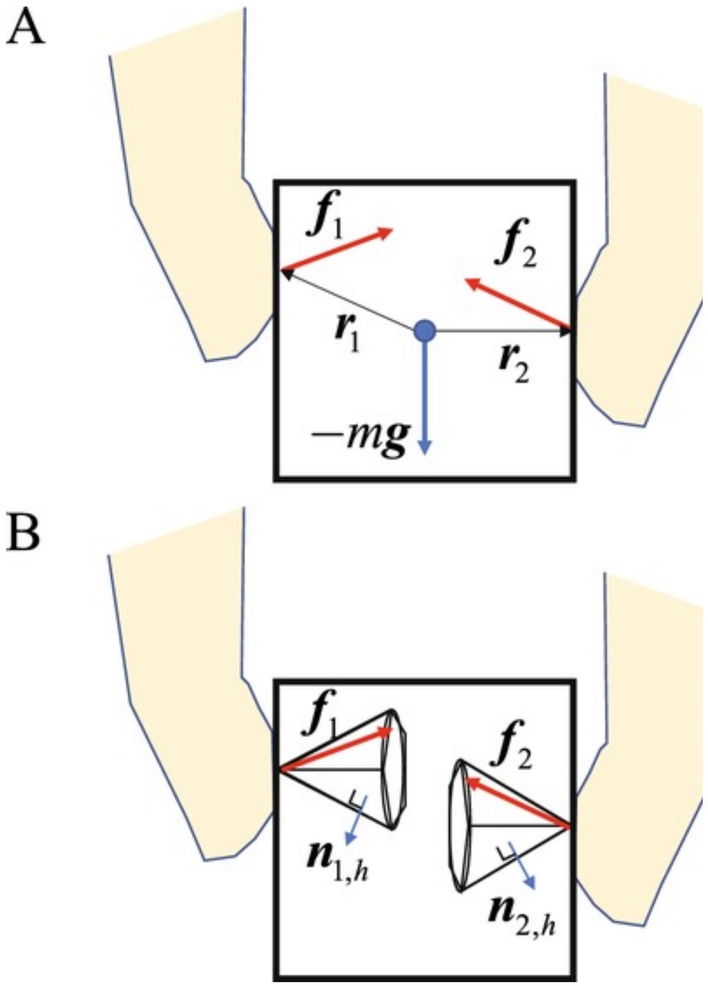


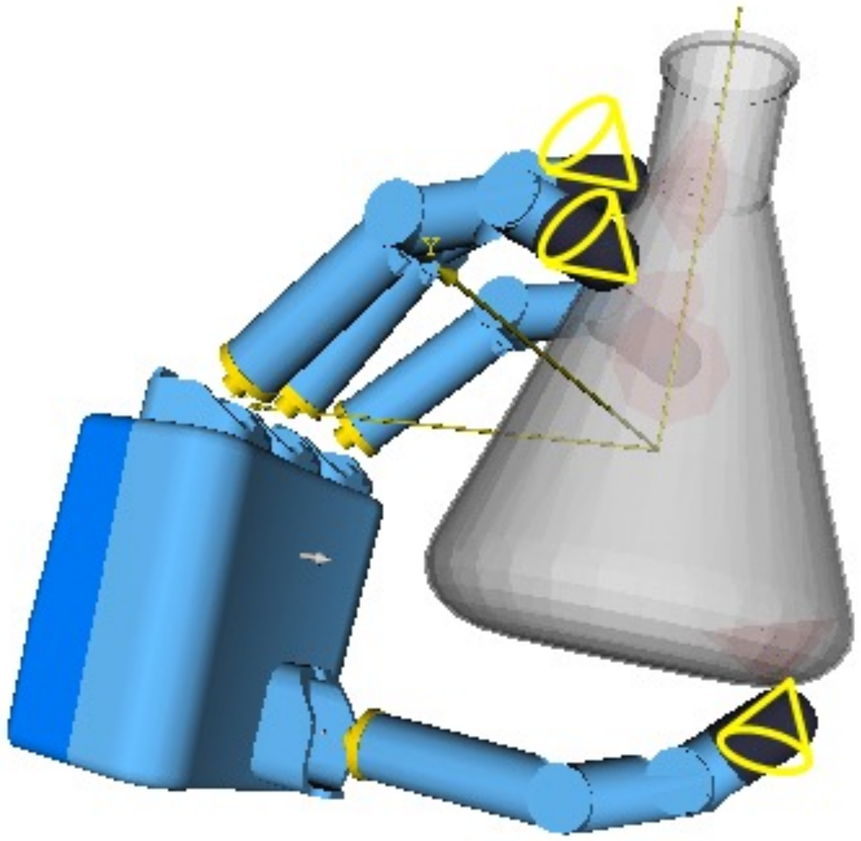
Embodiment Gap

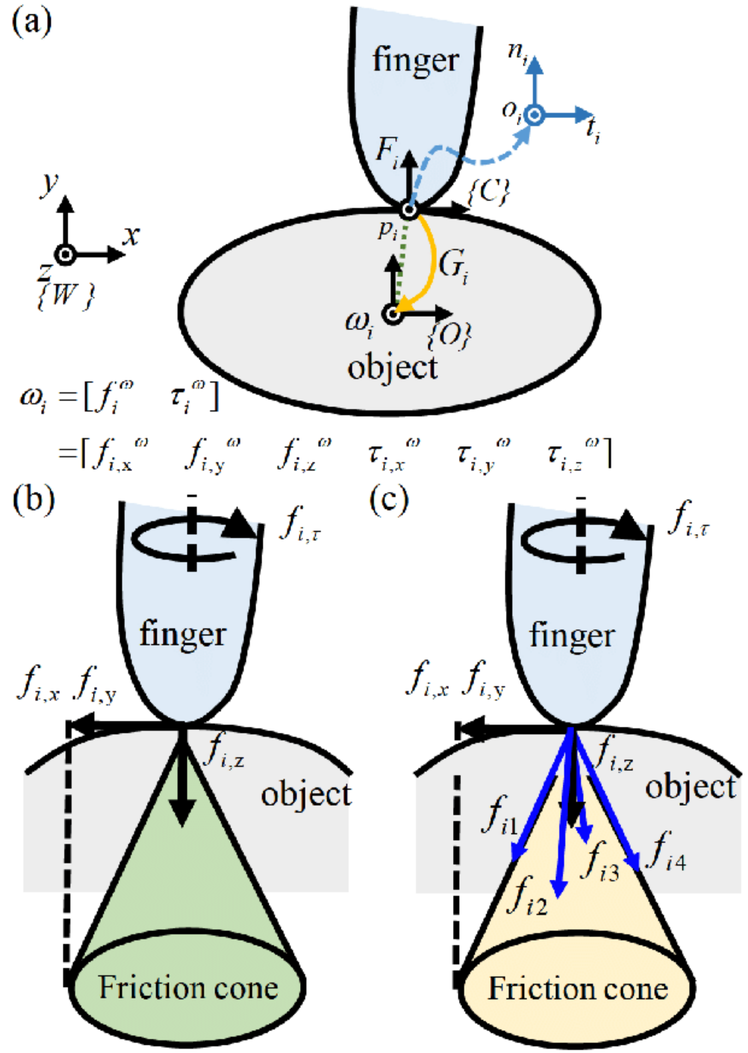


Interaction Mechanics Gap

“Textbook” Analysis of Contact







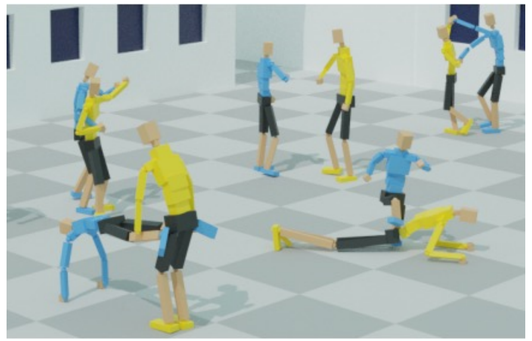
Google DeepMind

GAZEBO

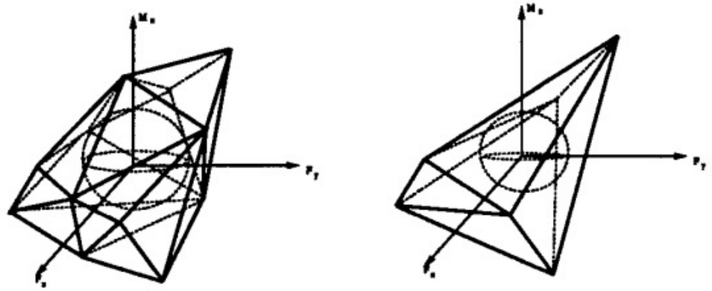
MuJoCo

Advanced physics simulation

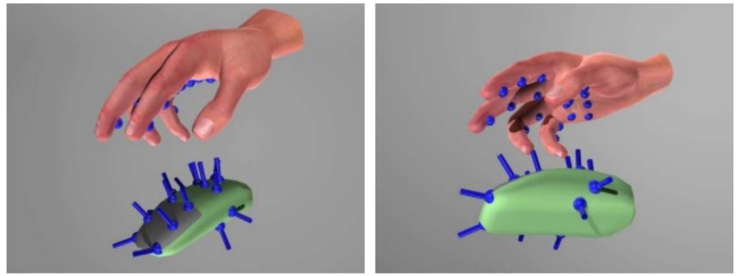
Very Useful Applications!



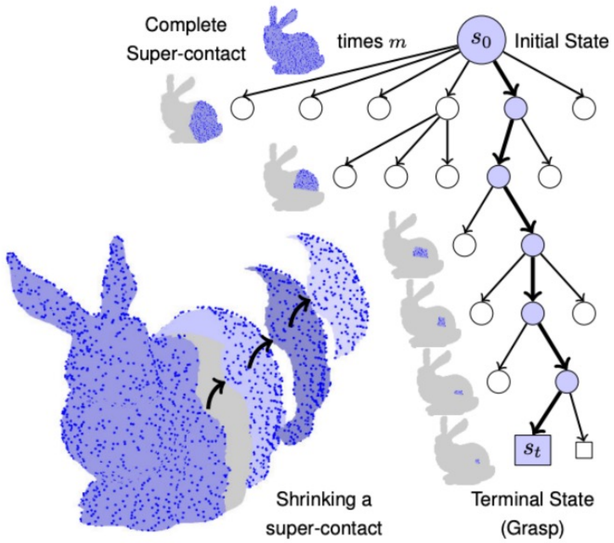
[Zhang et. al, 2023]



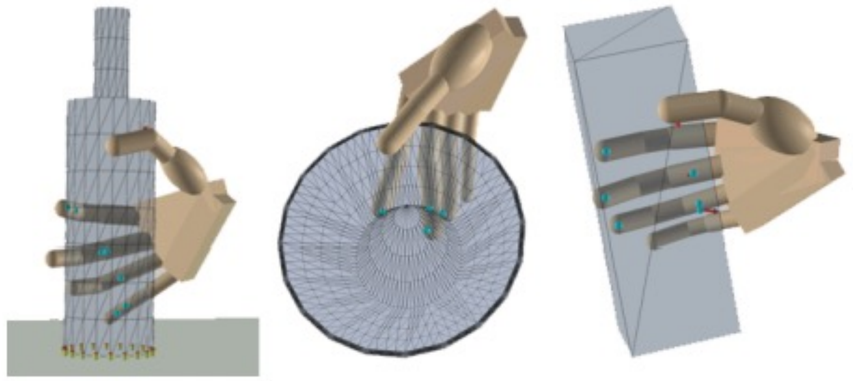
[Ferrari & Canny, 1992]



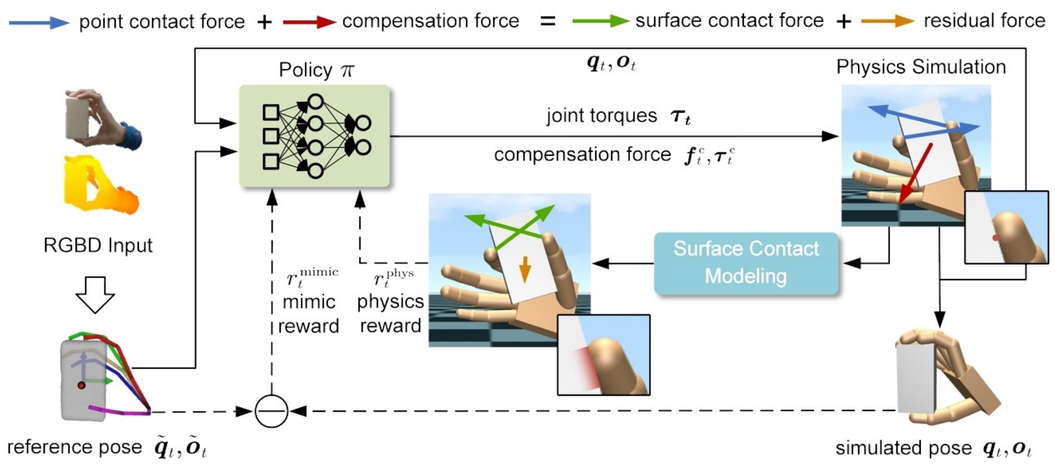
[Li et. al, 2007]



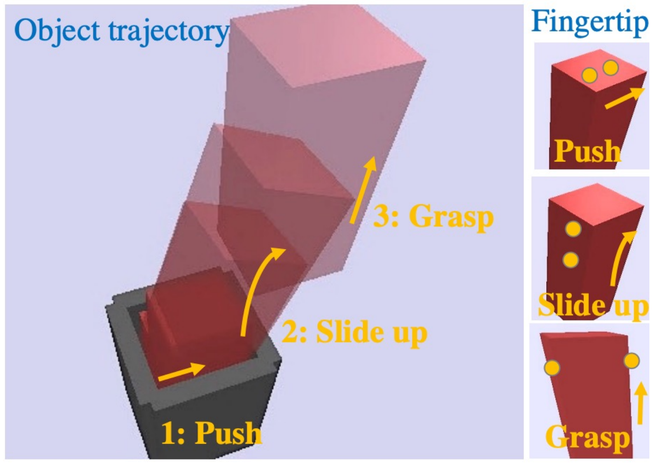
[Hang et. al, 2017]



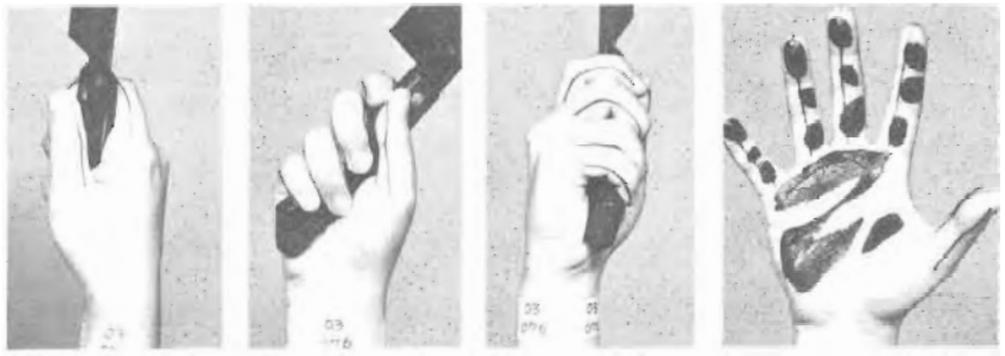
[Ye & Liu, 2012]



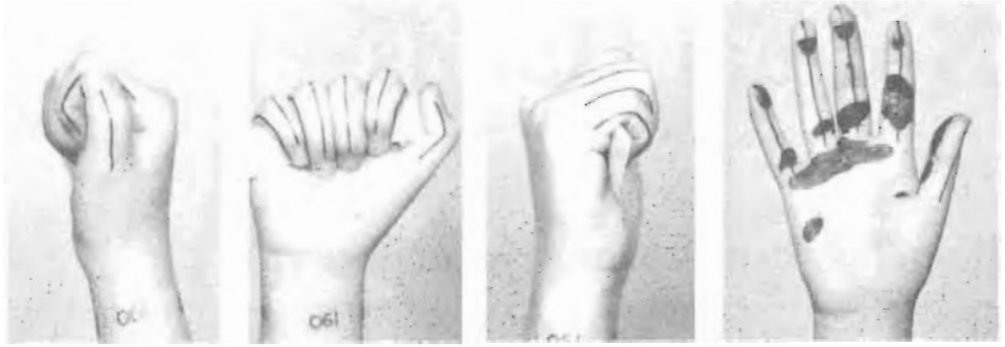
[Hu et. al, 2024]



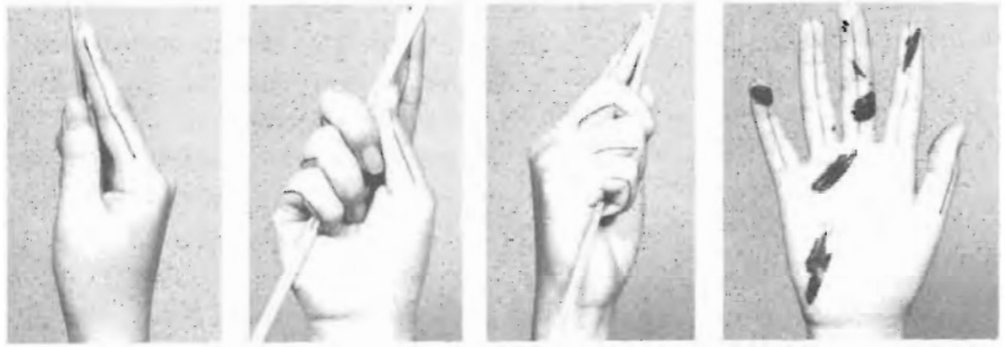
[Cheng et. al, 2021]



a. Power grip - Standard type (PoS)



b. Power grip - Hook type (PoH)



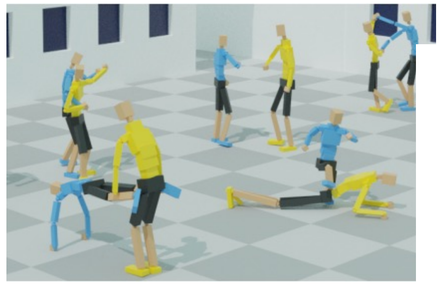
c. Power grip - Index Finger Extension type (PoI)

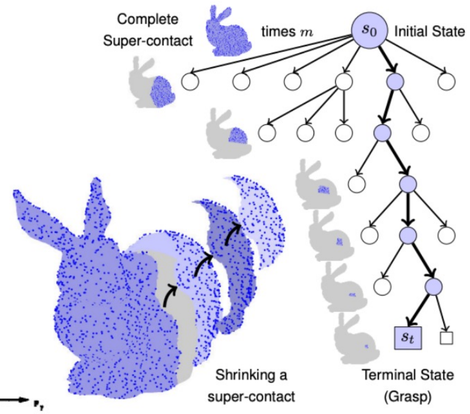
[Kamakura et. al, 1980]

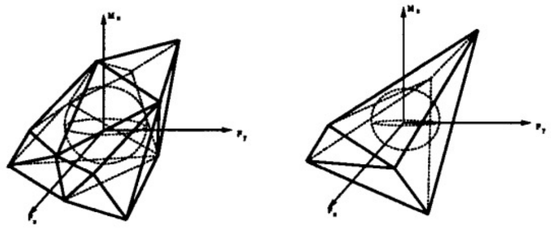
[Brahmbhatt et. al, 2019]

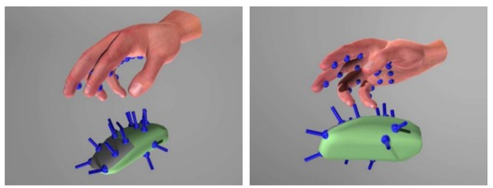
Ultimate Goal

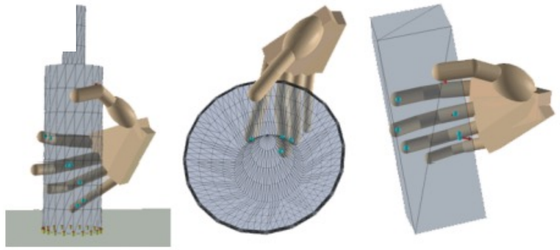
Do All of This

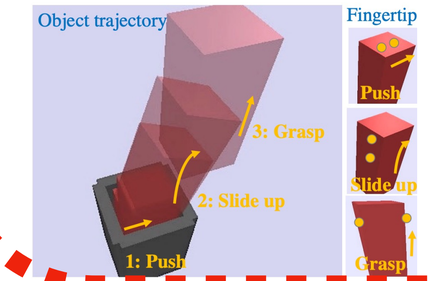


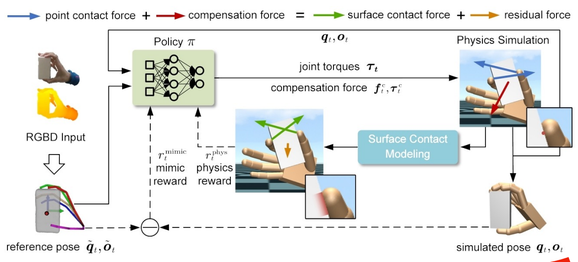








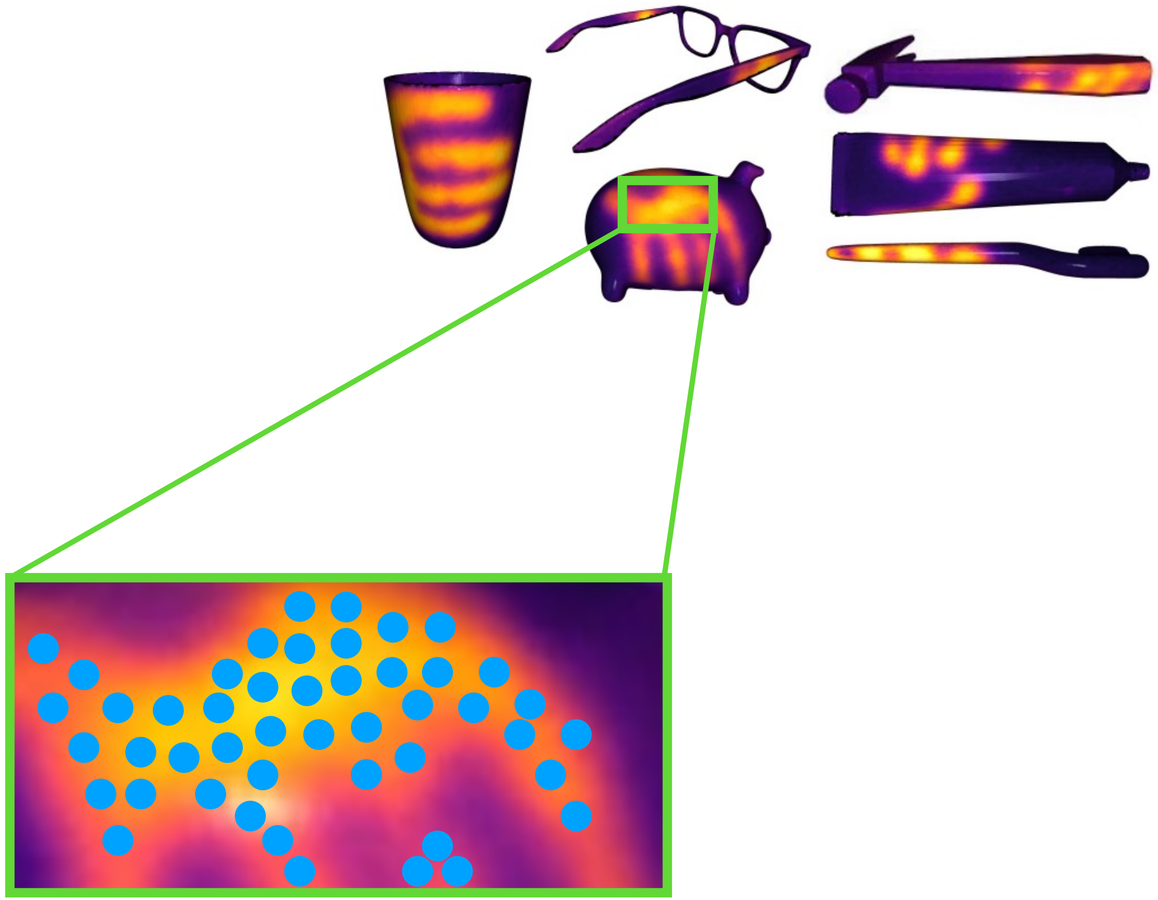




With This


Ok...
Good
OFFICIAL
RBU
LEAGUE

What Can’t We Just Treat Areas as a Bunch of Points?

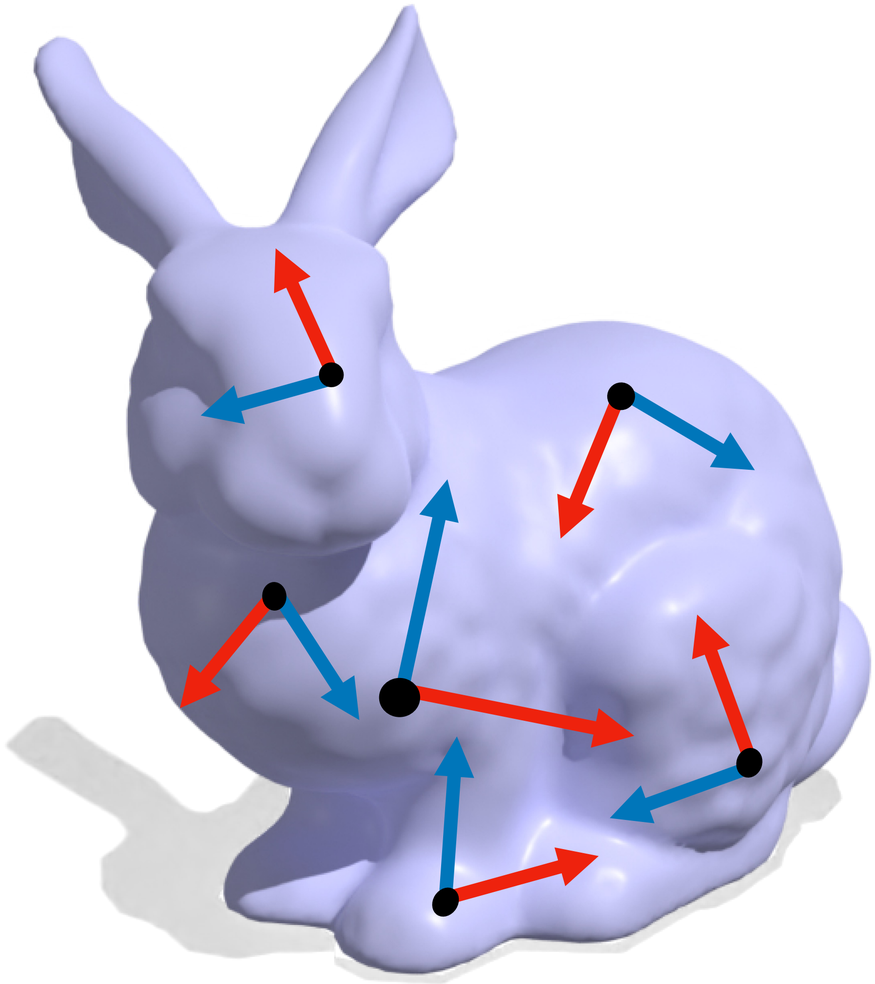


✓ **Single Point Analysis Techniques**

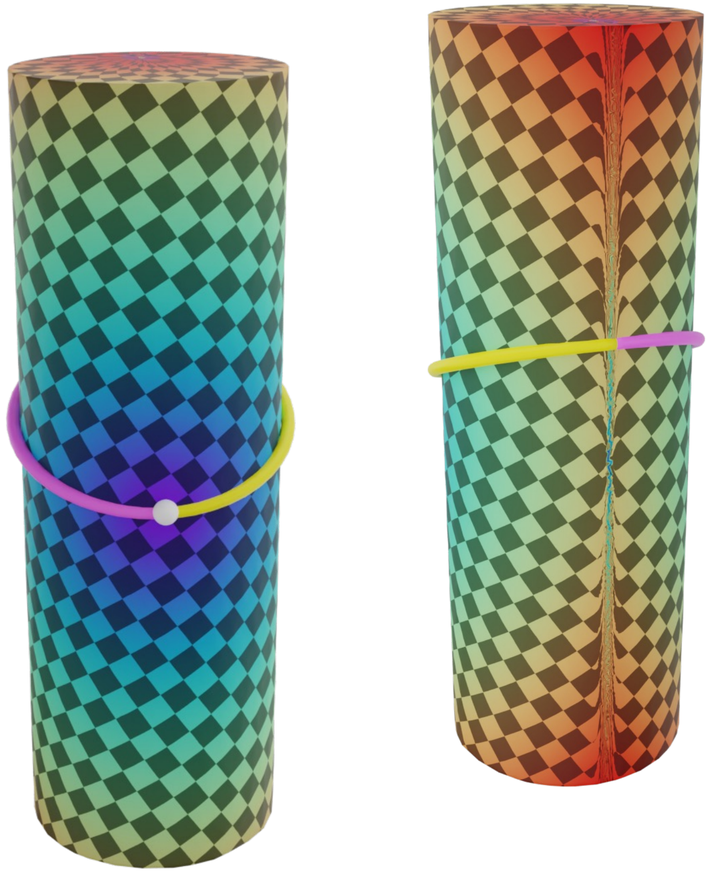
~ **Point-Wise Planning**

✗ **Transform as a Single Entity**

Complexities of Surfaces

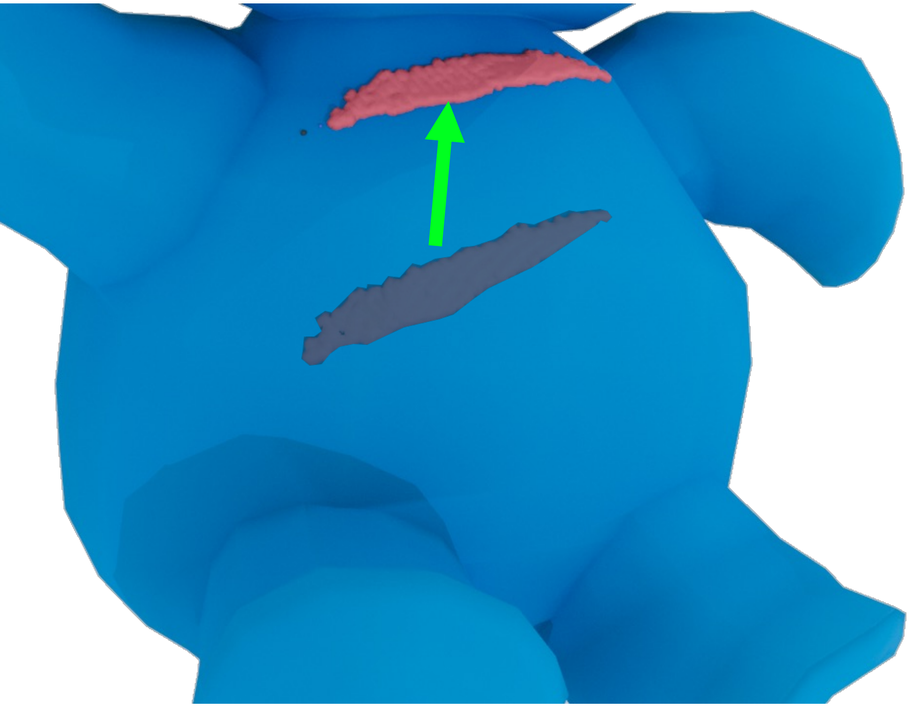


No Global Coordinate System



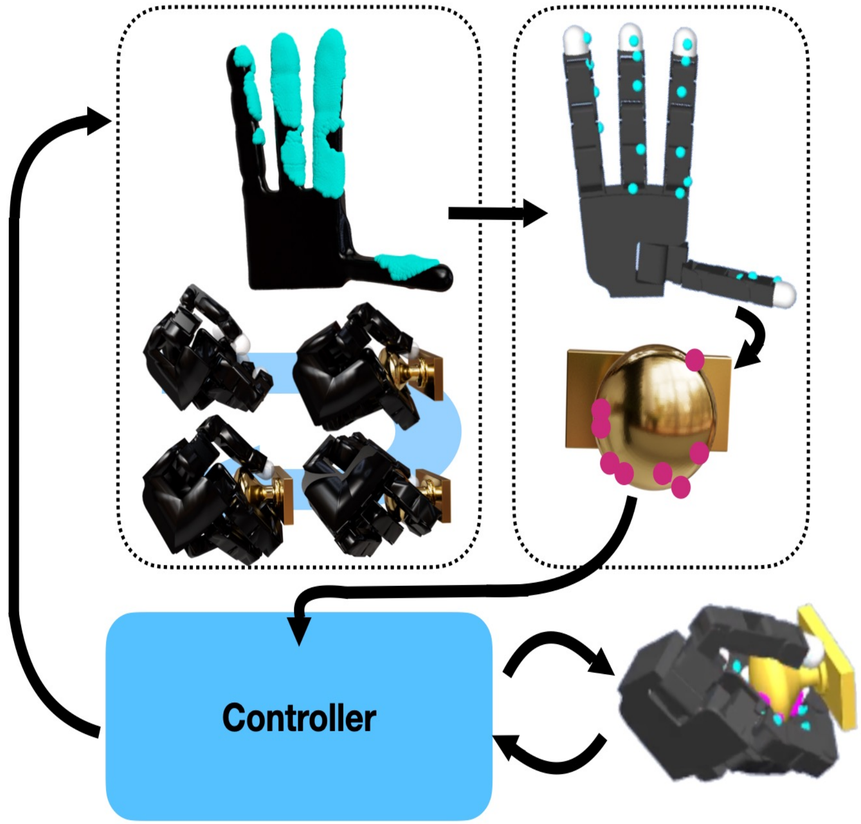
Large Path Discontinuities

What Do We Want to Do?



Transform

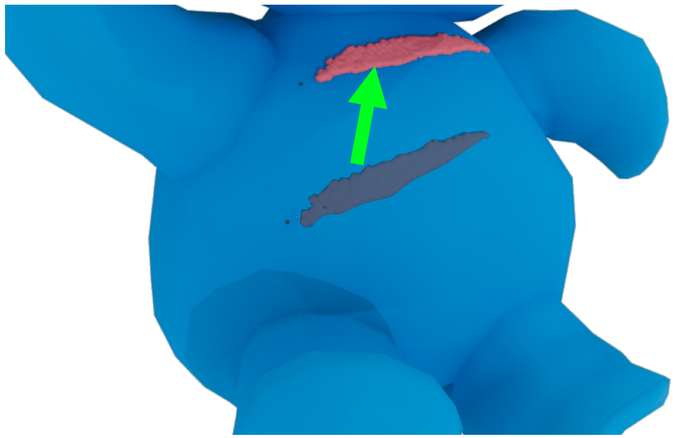
Extract Correspondences



Control

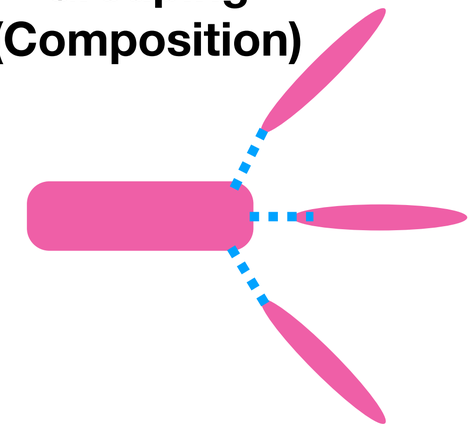
What Operations Should We Support?

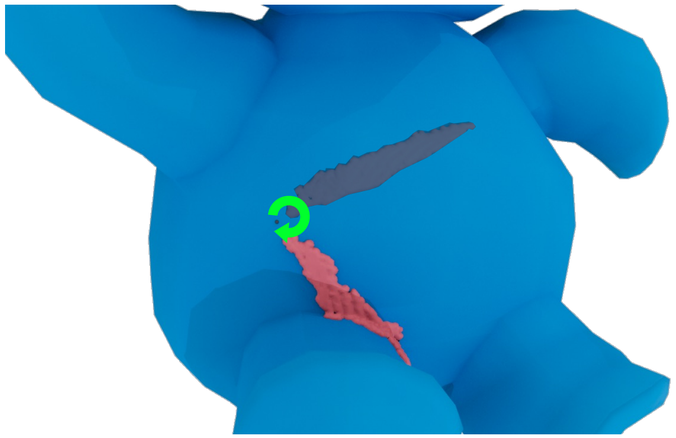
Relocation (Translation)



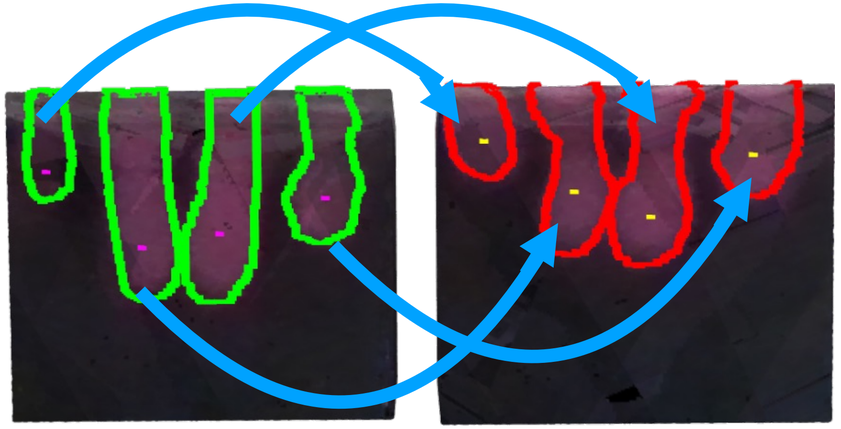
Bending ("Isometric" Deformation)

Grouping (Composition)



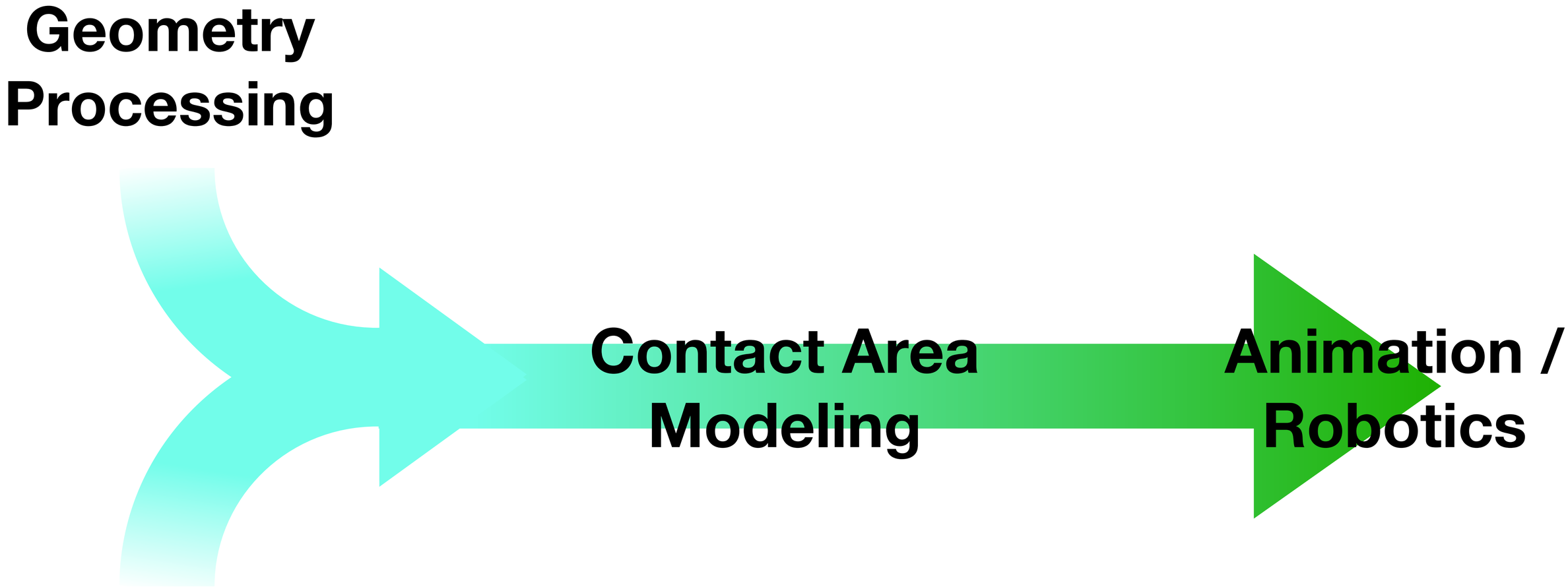


Reorientation (Rotation)



Warping (Non-Isometric Deformation)

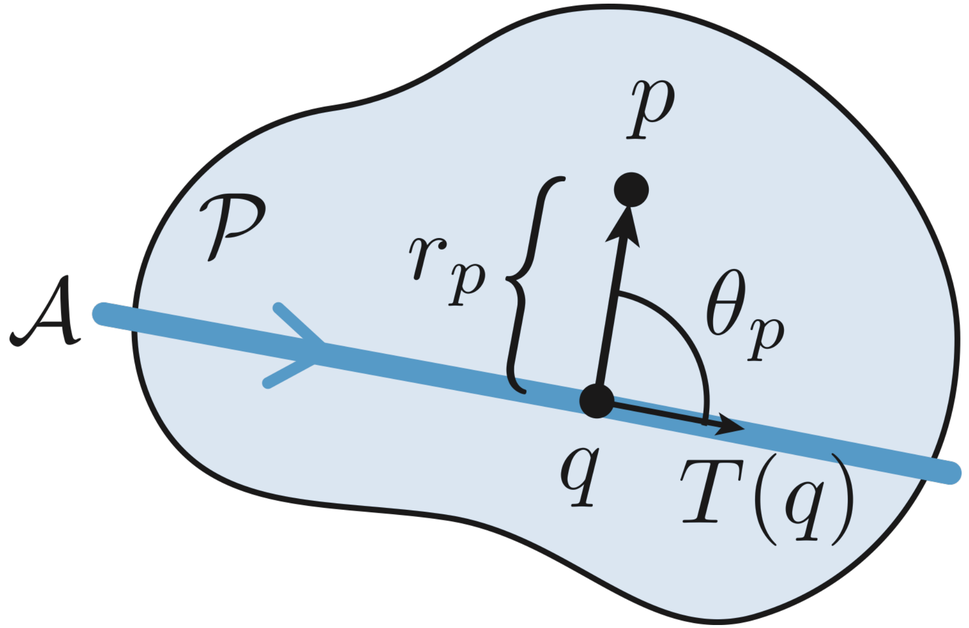
Correspondance (Transfer)


Geometry Processing
Interaction Mechanics
Contact Area Modeling
Animation / Robotics

Three Models

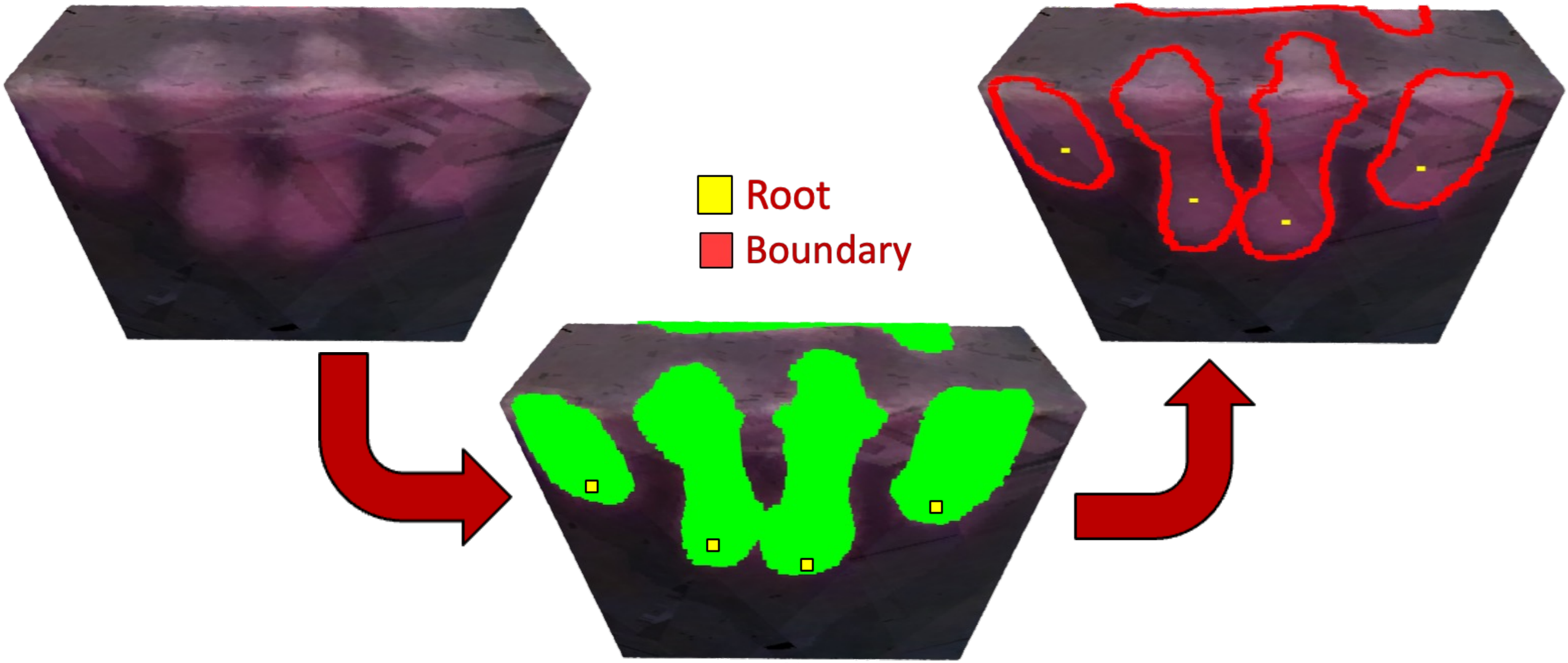
Boundary Model

Single-Point-Embedded (SPE) Model

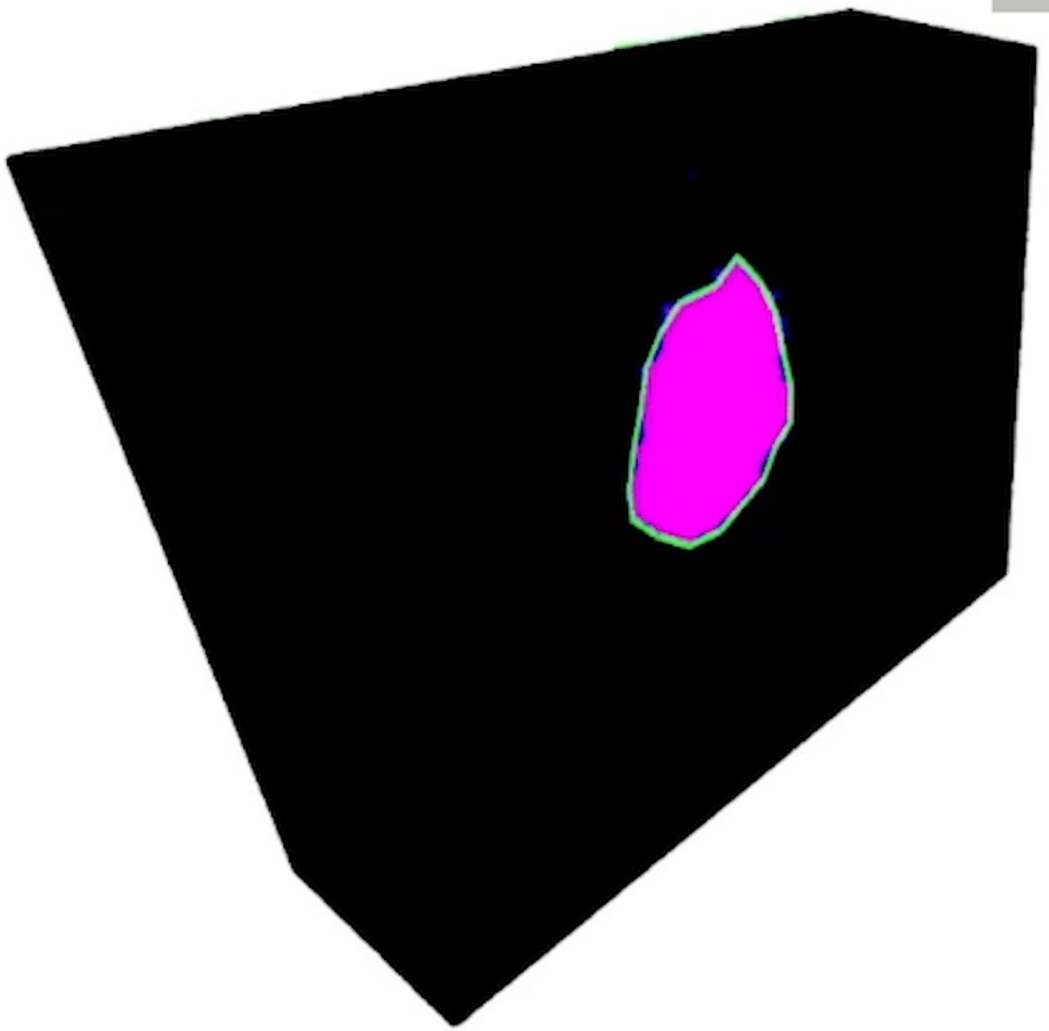


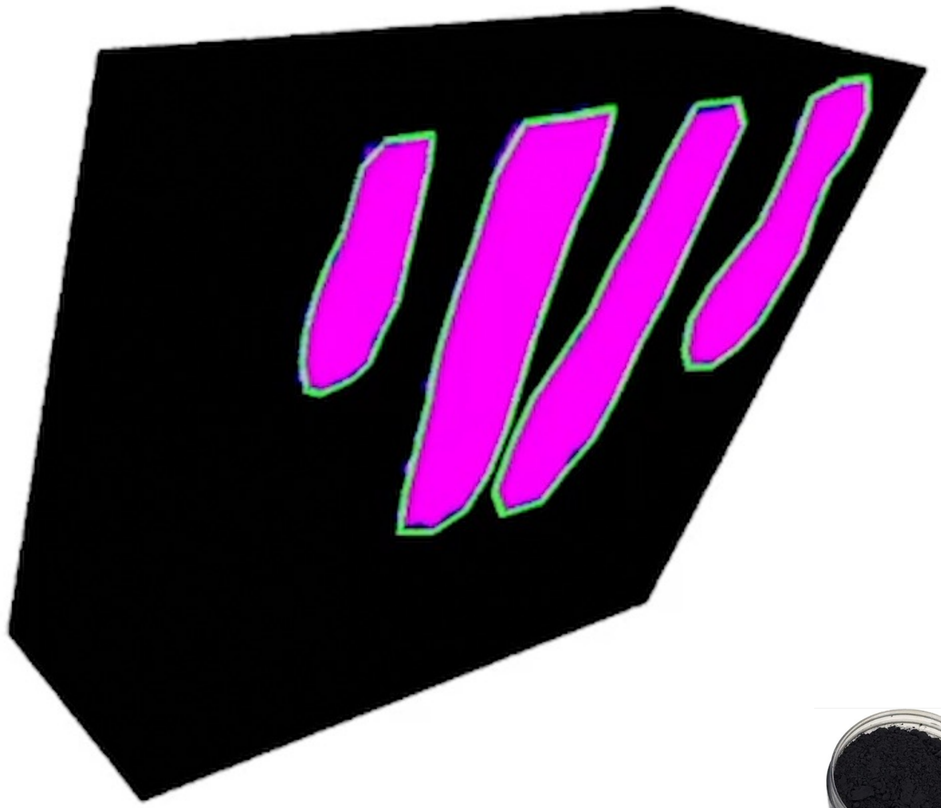
Axis-Embedded (AE) Model

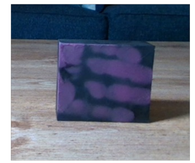
Boundary Model



Contact Tracing: A Low Cost Reconstruction Framework for Surface Contact Interpolation [IROS 2021]

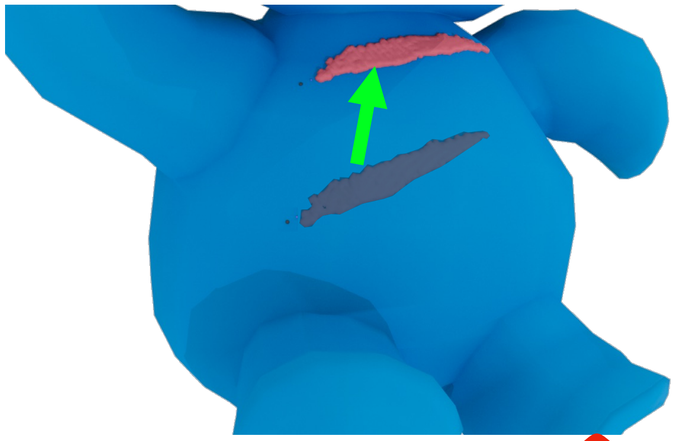


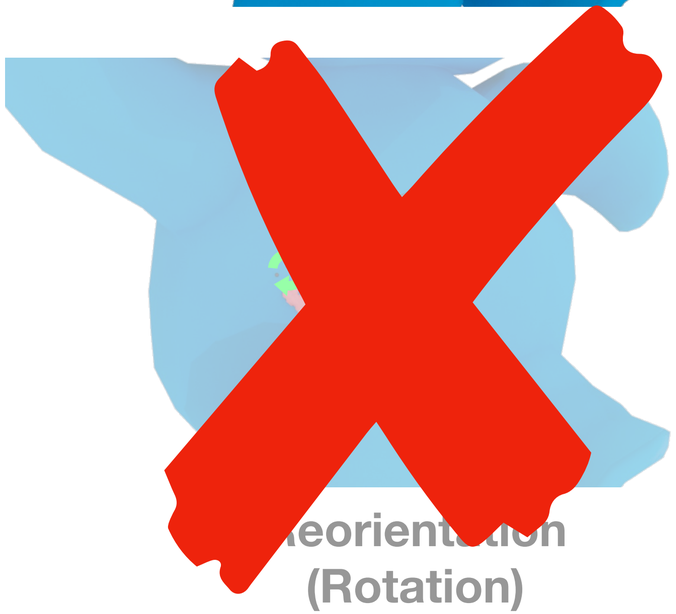




Limitations

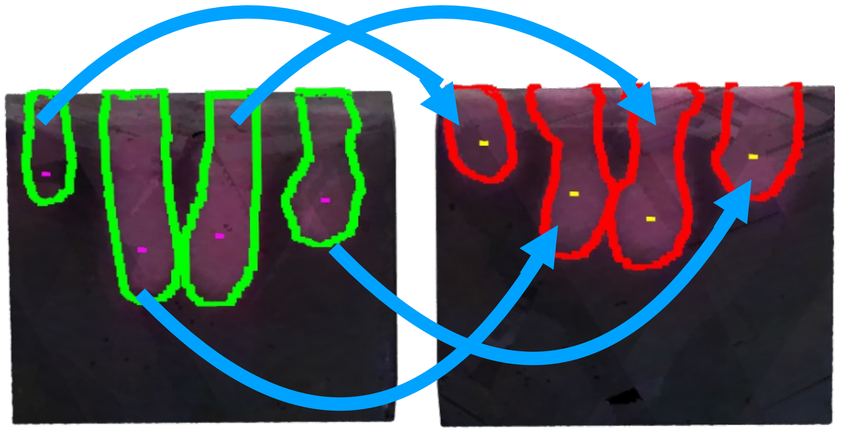
Relocation (Translation)





Reorientation (Rotation)

Bending ("Isom… Defor…)

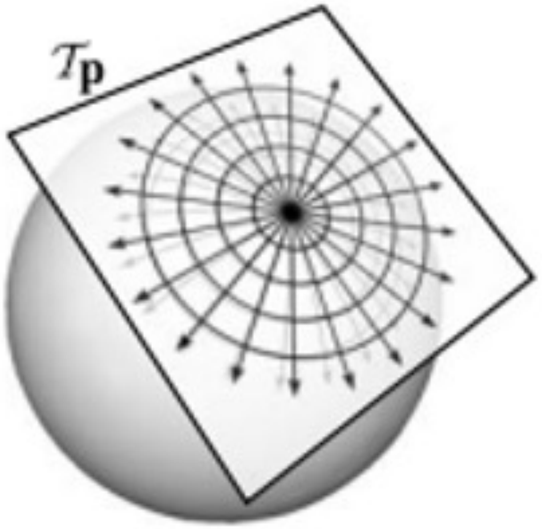


Warping (Non-Isometric Deformation)

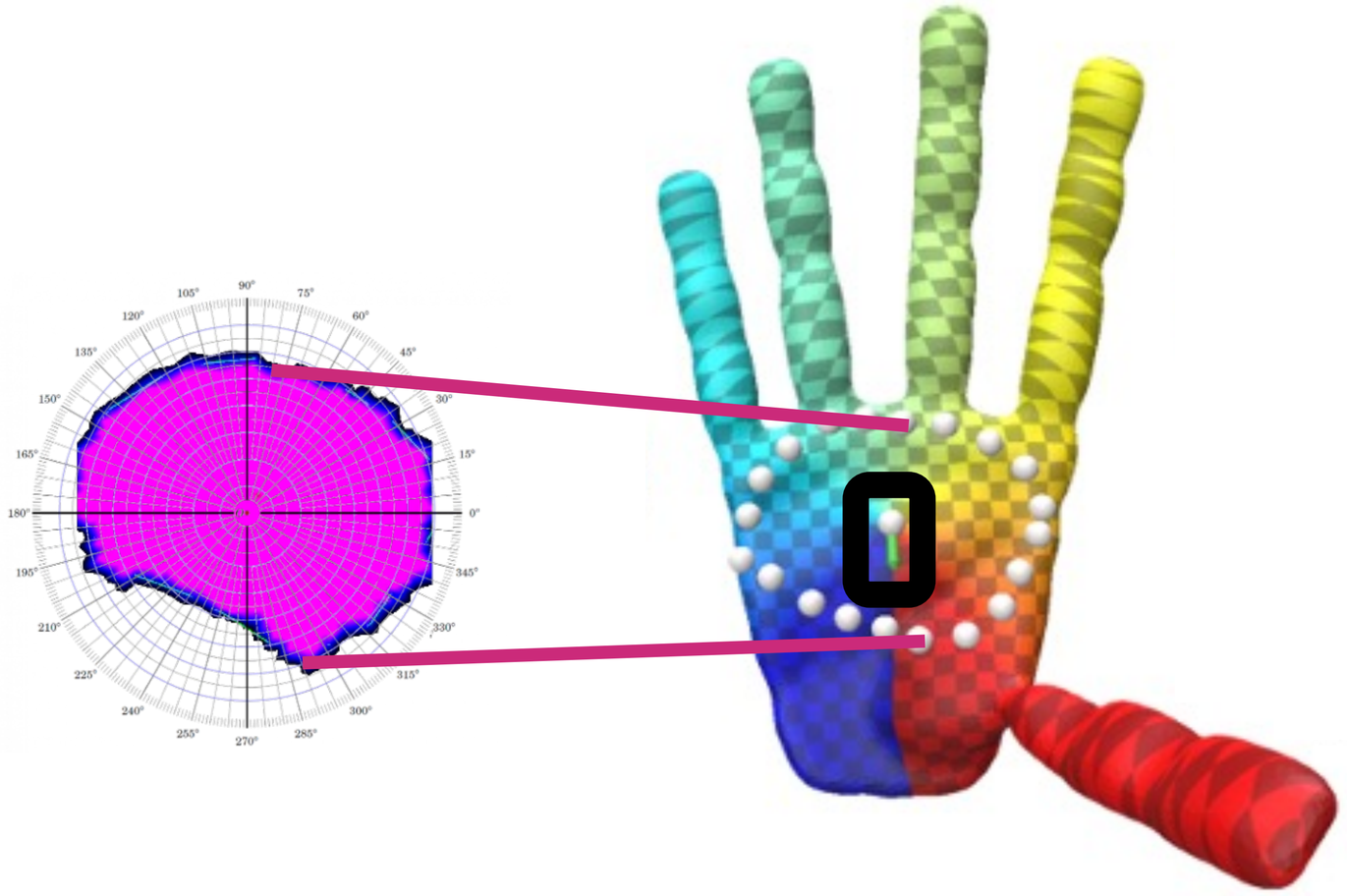
Grouping (C…osition)

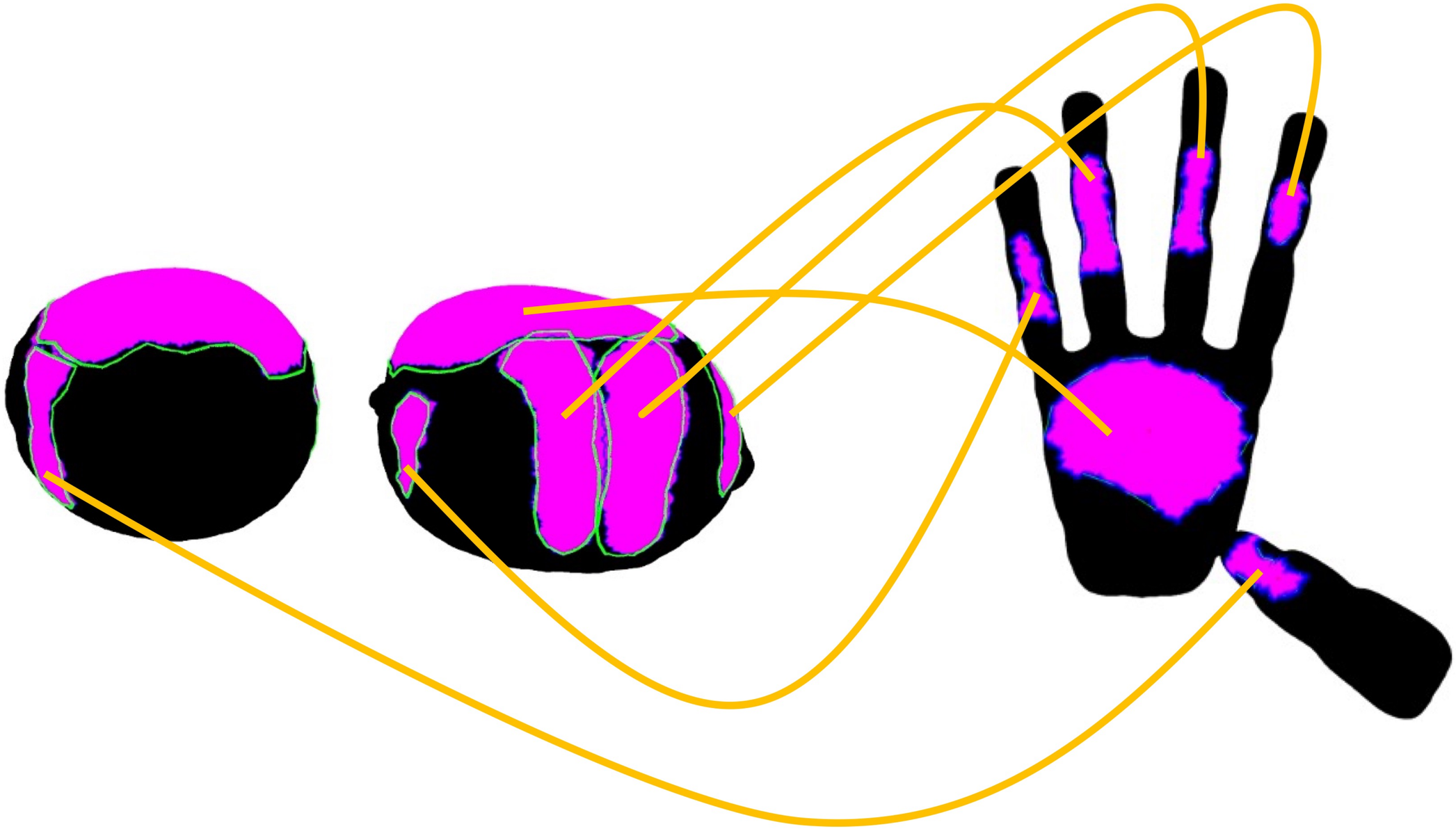
Correspondance (Transfer)

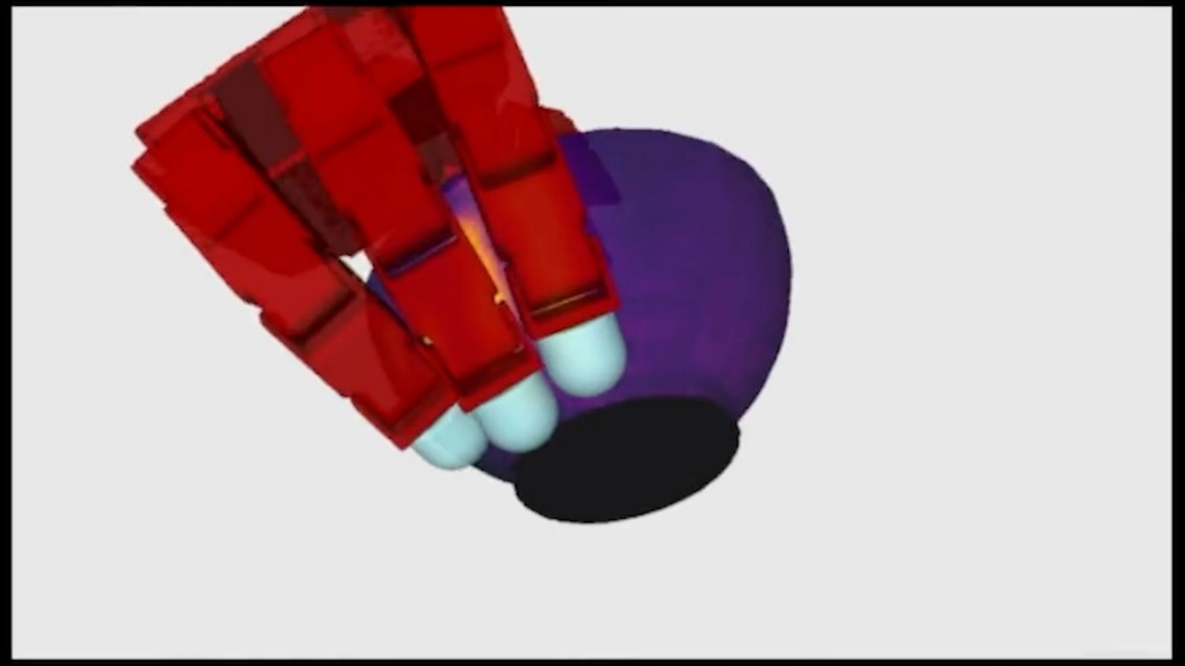
Single Point Embedding Model

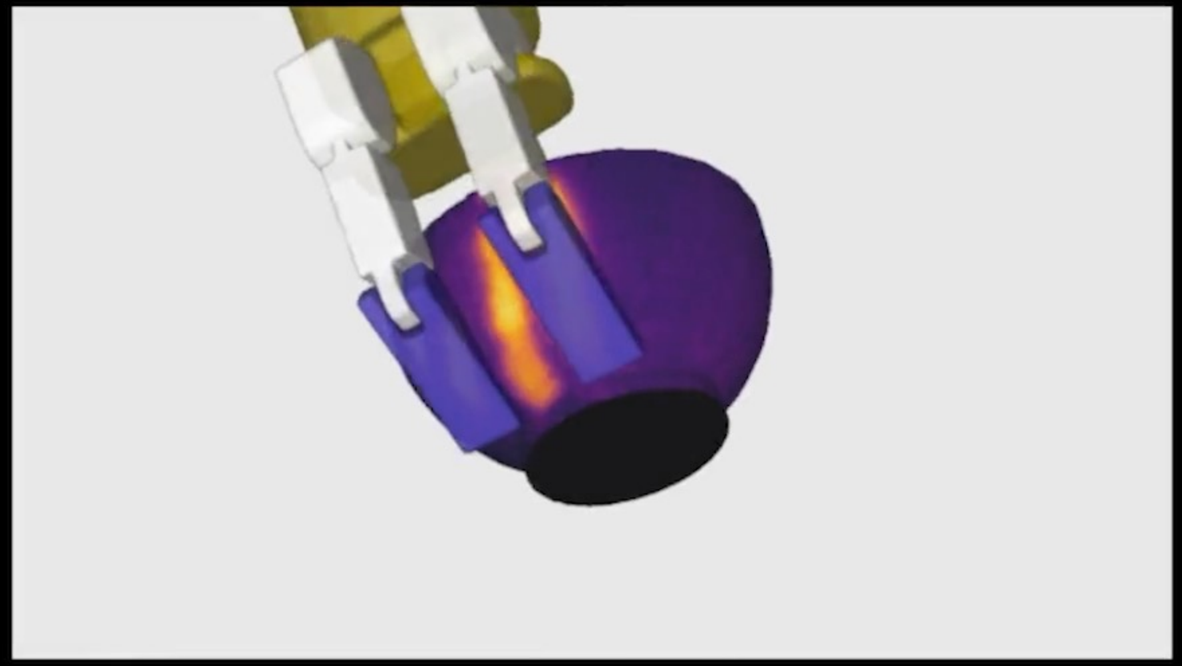


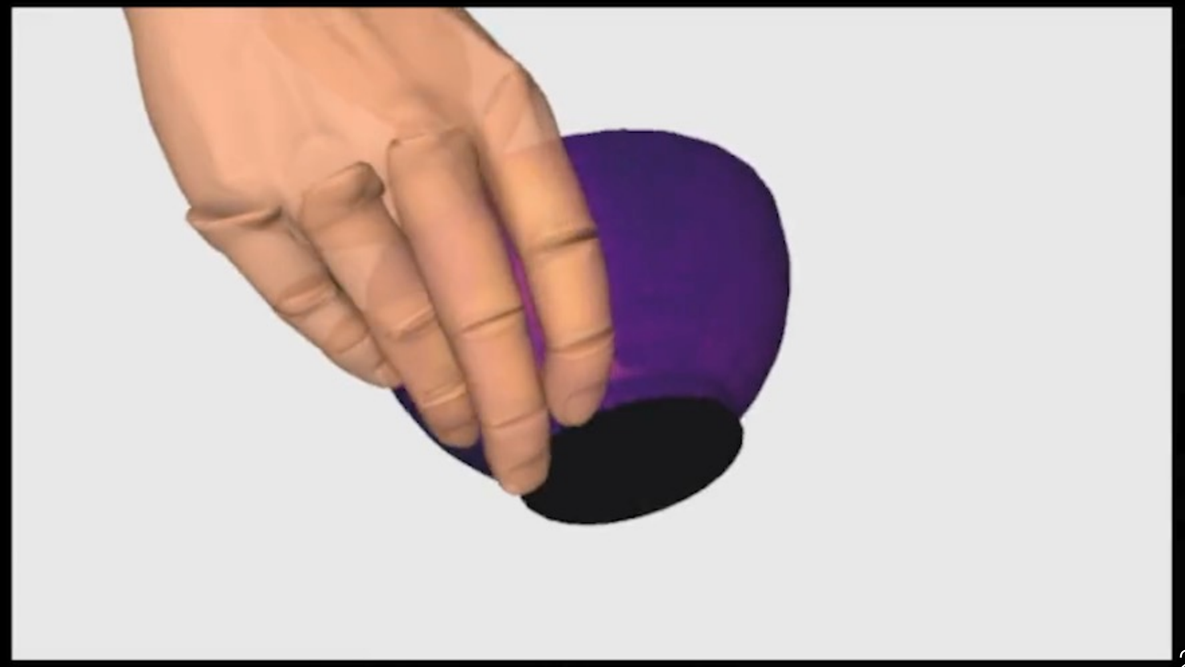
[Schmidt et. al, 2006]





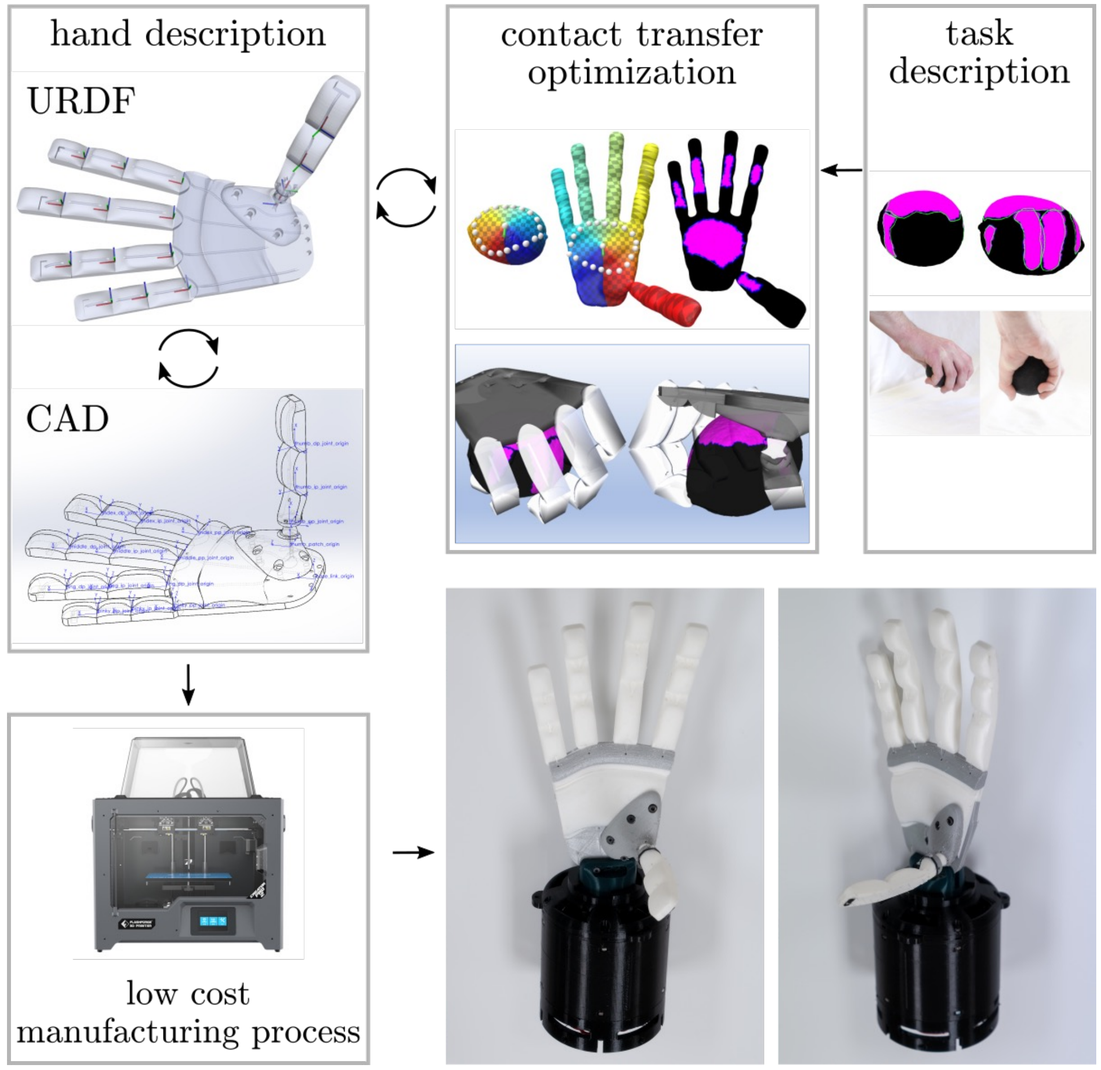






Application: Rapid Prototyping

Towards Very Low-Cost Iterative Prototyping for Fully Printable Dexterous Soft Robotic Hands [Robosoft 2022]



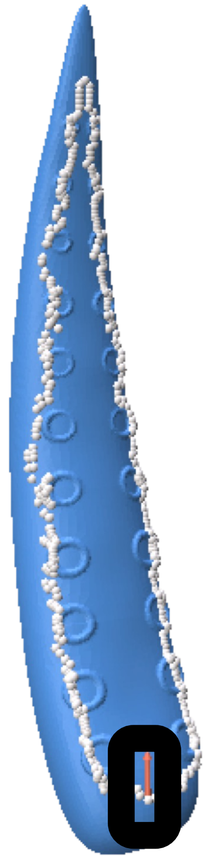
In Collaboration With:

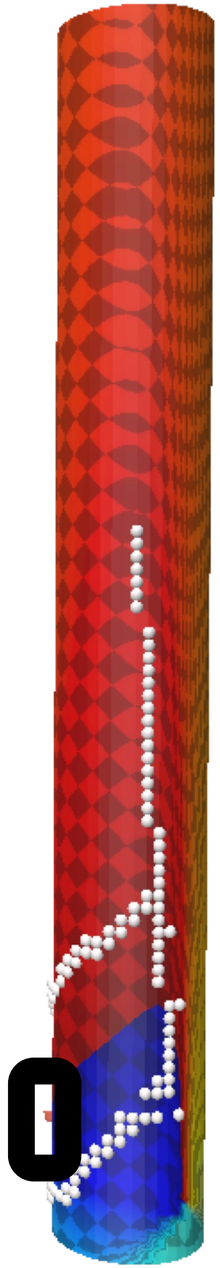
Limitations

$$c_i^* = \arg\min_{y} \quad \| log_x(y) - log_x(c_i) \|_2^2$$
$$\text{s.t.} \quad log_x(\cdot) \quad = \quad f(x; \vec{x})$$

Time Complexity: O(CN)

N: Total Number of Elements
C: Total Number of Contacts





Performance Problems

Robustness Problems

Axis-Embedded Model

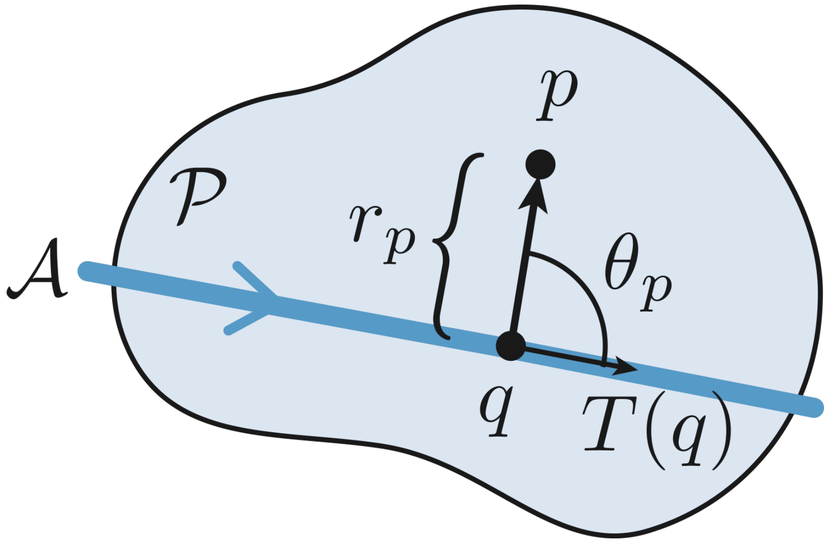
$exp_q(p) := T_q(r_p, \theta_p)$

Exponential map
coordinates of p
in tangent basis of q

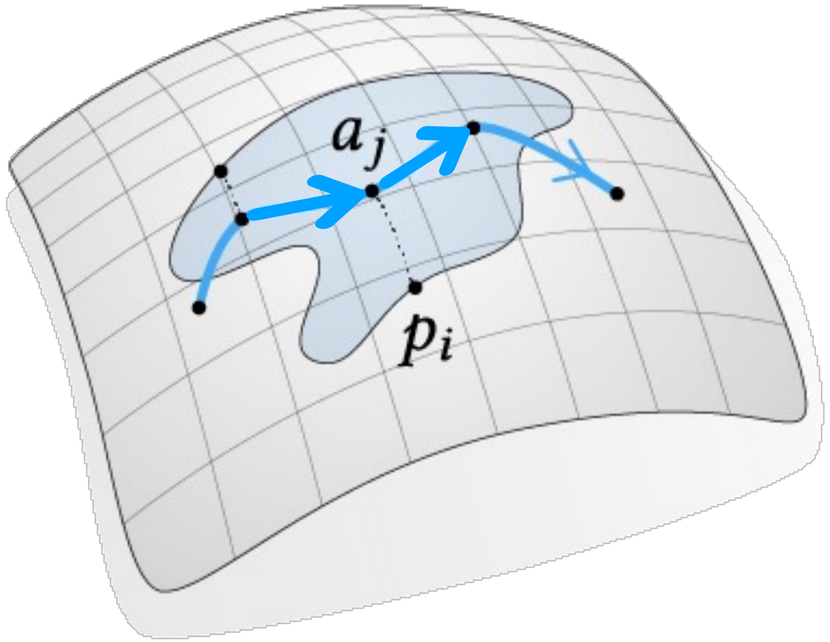
$\{a_0, \ldots, a_M\} \in Axis$

$a_{j+1} = a_j + (d_j, \phi_j)$

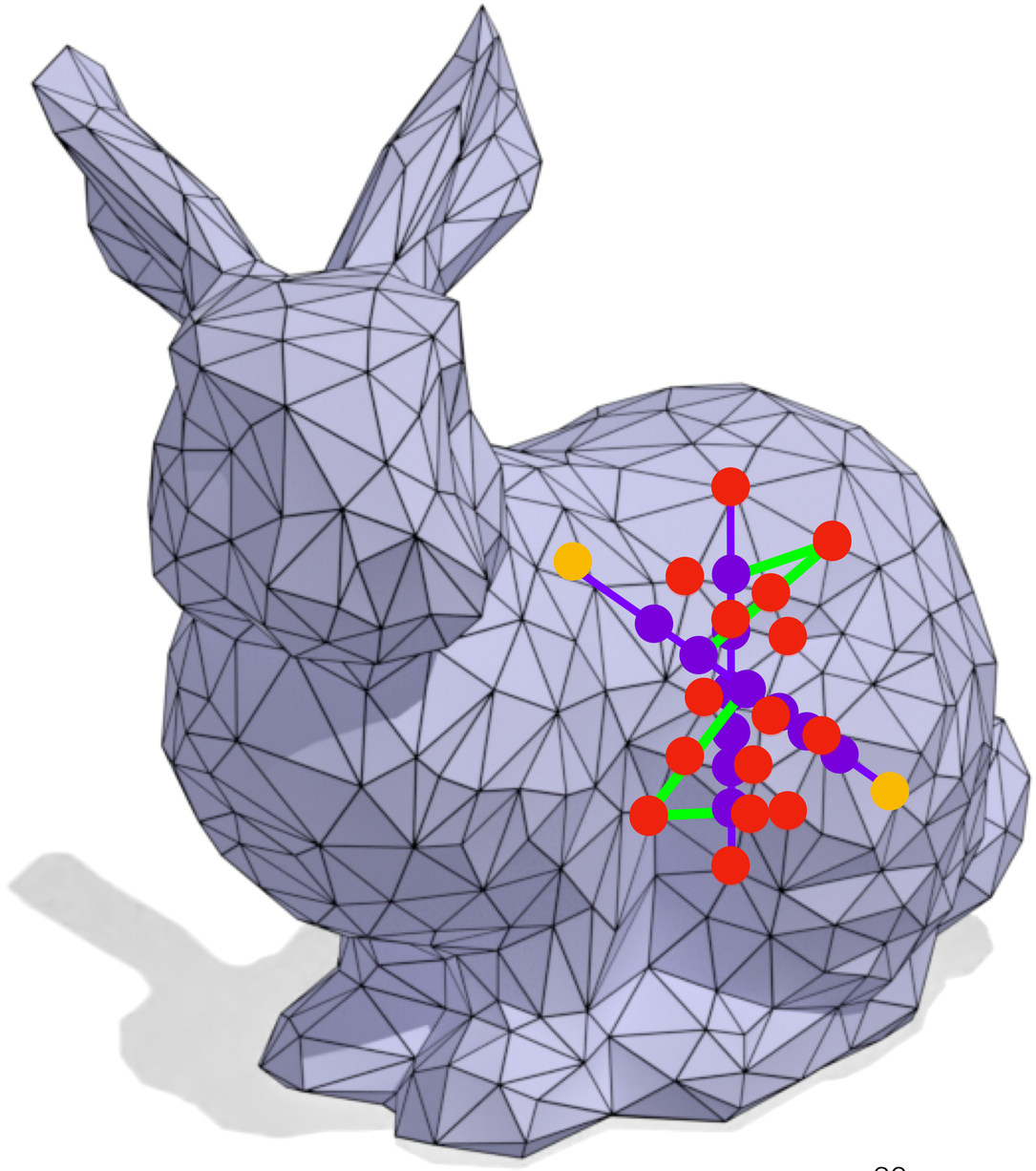
d_j := distance
ϕ_j := turning angle



$q = a_j^*$



AE Model Parameterization



The Vector Heat Method

NICHOLAS SHARP, YOUSUF SOLIMAN, and KEENAN CRANE, Carnegie Mellon University

This paper describes a method for efficiently computing parallel transport of tangent vectors on curved surfaces, or more generally, any vector-valued data on a curved manifold. More precisely, it extends a vector field defined over any region to the rest of the domain via parallel transport along shortest geodesics. This basic operation enables fast, robust algorithms for extrapolating level set velocities, inverting the exponential map, computing geometric medians and Karcher/Fréchet means of arbitrary distributions, constructing centroidal Voronoi diagrams, and finding consistently ordered landmarks. Rather than evaluate parallel transport by explicitly tracing geodesics, we show that it can be computed via a short-time heat flow involving the *connection Laplacian*. As a result, transport can be achieved by solving three prefactored linear systems, each akin to a standard Poisson problem. To implement the method we need only a discrete connection Laplacian, which we describe for a variety of geometric data structures (point clouds, polygon meshes, *etc.*). We also study the numerical behavior of our method, showing empirically that it converges under refinement, and augment the construction of intrinsic Delaunay triangulations (iDT) so that they can be used in the context of tangent vector field processing.

CCS: • **Mathematics of computing** → **Discretization**; **Partial differential equations**; • **Computing methodologies** → **Shape analysis**;

Additional Key Words and Phrases: discrete differential geometry, parallel transport, velocity extrapolation, logarithmic map, exponential map, Karcher mean, geometric median

ACM Reference Format:
Nicholas Sharp, Yousuf Soliman, and Keenan Crane. 2019. The Vector Heat Method. *ACM Trans. Graph.* 38, 3, Article 00 (June 2019), 19 pages. https://doi.org/00.0000/0000000.0000000

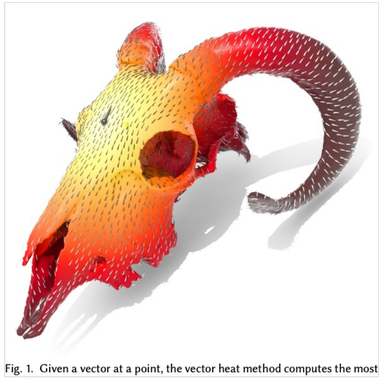


Fig. 1. Given a vector at a point, the vector heat method computes the most parallel vector at every other point. The method easily generalizes to other data (such as a velocity field along a curve), providing a novel and efficient way to implement fundamental algorithms across geometry and simulation.

1 INTRODUCTION

Given a vector at a point of a curved domain, how do we find the most parallel vector at all other points (as shown in Fig. 1)? This "most parallel" vector field—not typically considered in numerical algorithms—provides a surprisingly valuable starting point for a wide variety of tasks across geometric and scientific computing, from extrapolating level set velocity to computing centers of distributions. To compute this field, one idea is to transport the vector along explicit paths from the source x to all other points y, but even just constructing these paths is already quite expensive (Sec. 2). We instead leverage a little-used relationship between parallel transport and the *vector heat equation*, which describes the diffusion of a given vector field over a time t. As t goes to zero, the diffused field is related to the original one via parallel transport along minimal geodesics, *i.e.*, shortest paths along the curved domain (Sec. 3.4).

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https://doi.org/00.0000/0000000.0000000

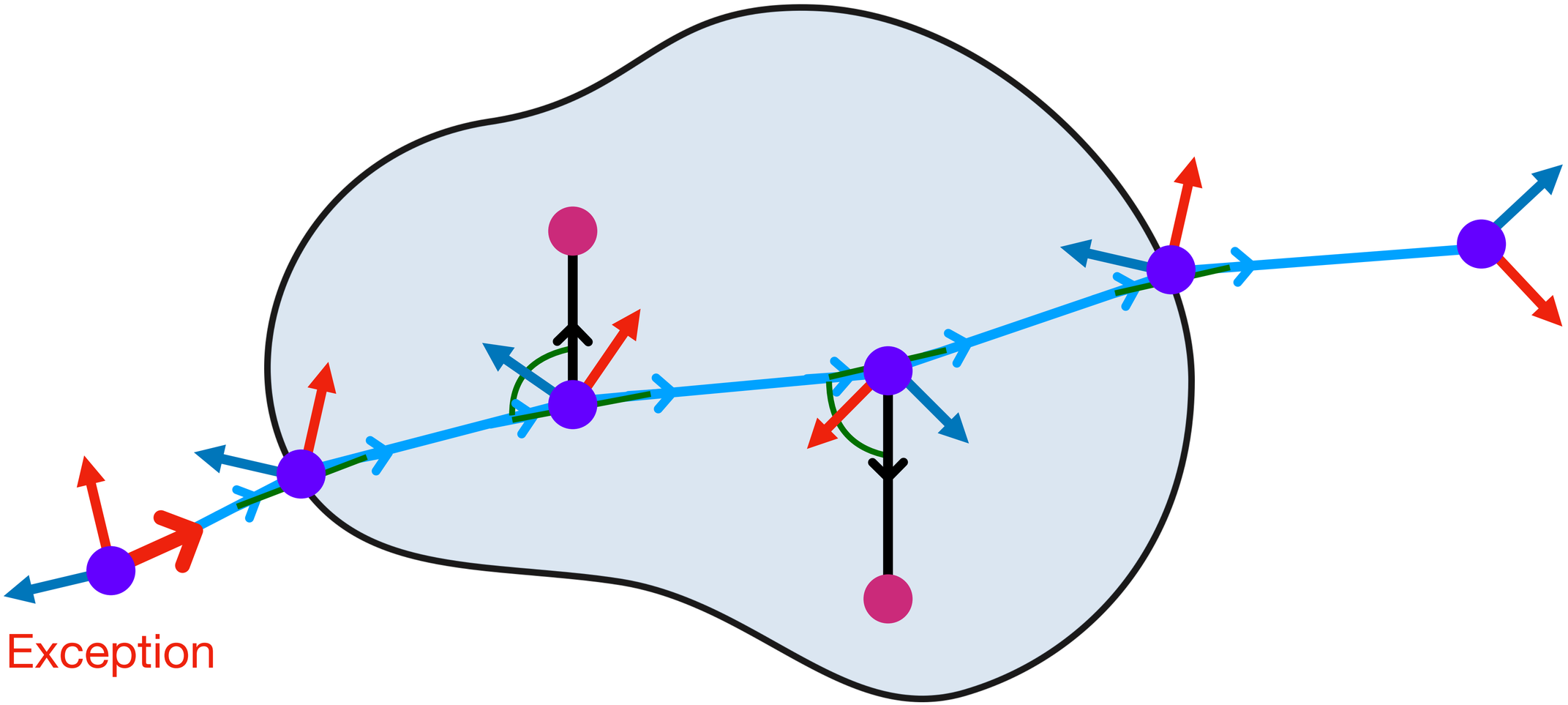
The same principle applies not only to point sources, but also to vector fields over curves or other subsets of the domain. Since diffusion equations are expressed in terms of standard Laplace-like operators, we effectively reduce parallel transport tasks to sparse linear systems that are extremely well-studied in scientific computing—and can hence immediately benefit from mature, high-performance solvers. Moreover, since discrete Laplacians are available for a wide variety of shape representations (polygon meshes, point clouds, *etc.*), and generalize to many kinds of vector data (symmetric direction fields, differential forms, *etc.*), we can apply this same strategy to numerous applications. In particular, this paper introduces

- a fast method for computing parallel transport from a given source set (Sec. 4)
- an augmented intrinsic Delaunay algorithm for vector field processing (Sec. 5.4)
- the first method for computing a logarithmic map over the entire surface, rather than in a local patch (Sec. 8.2), and
- the first method for computing true Karcher/Fréchet means and geometric medians on general surfaces (Sec. 8.3).

We also describe how to discretize the connection Laplacian on several different geometric data structures and types of vector data (Sec. 6), and consider a variety of other applications including distance-preserving velocity extrapolation for level set methods, computing geodesic centroidal Voronoi tessellations (GCVT), and finding consistently ordered intrinsic landmarks (Sec. 8).

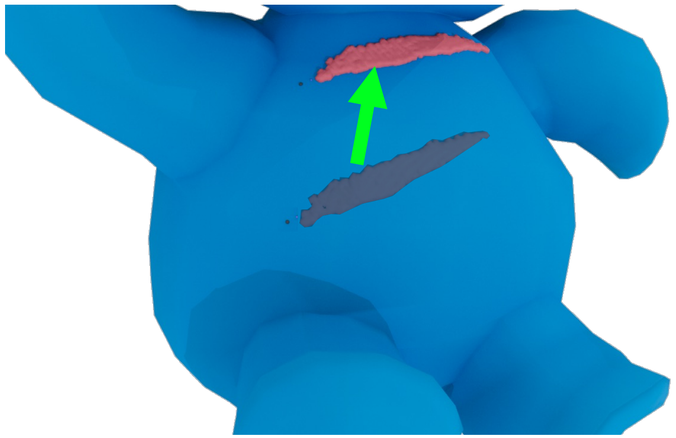
[Sharp & Crane, 2019]

More Details



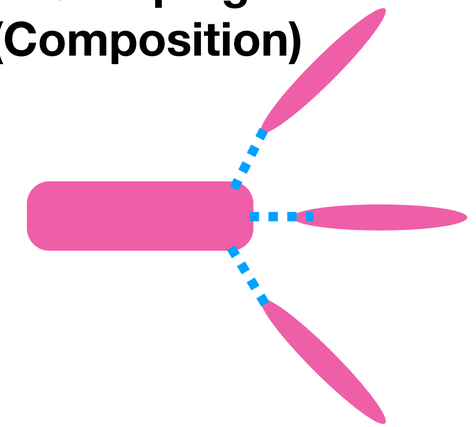
Supported Operations

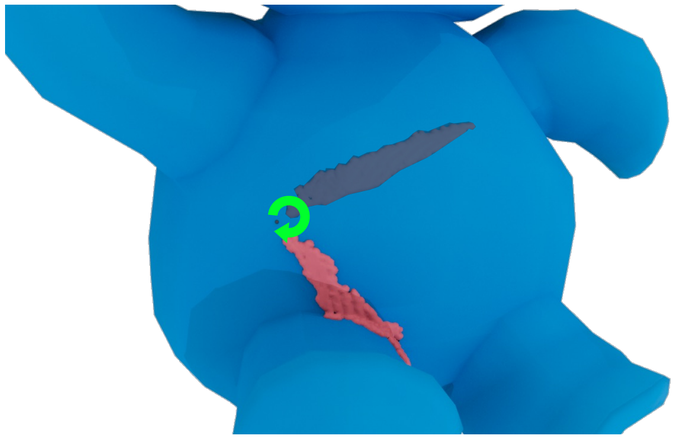
Relocation (Translation)



Bending ("Isometric" Deformation)

Grouping (Composition)



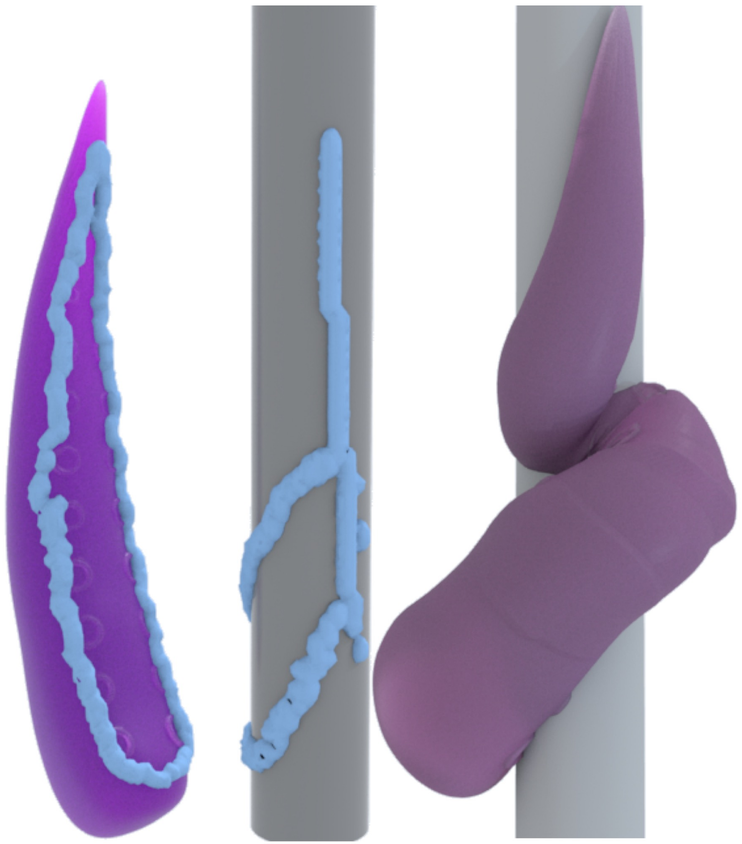


Reorientation (Rotation)

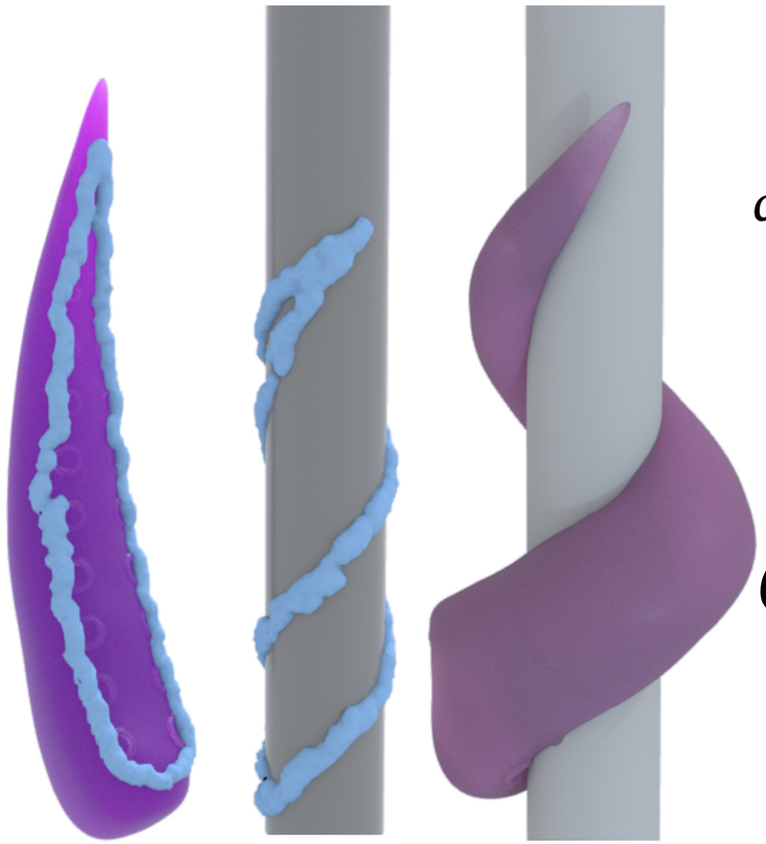
Warp
(Non-Isometric Deformation)

Correspondance (Transfer)

Robust and Fast



SPE Model



AE Model

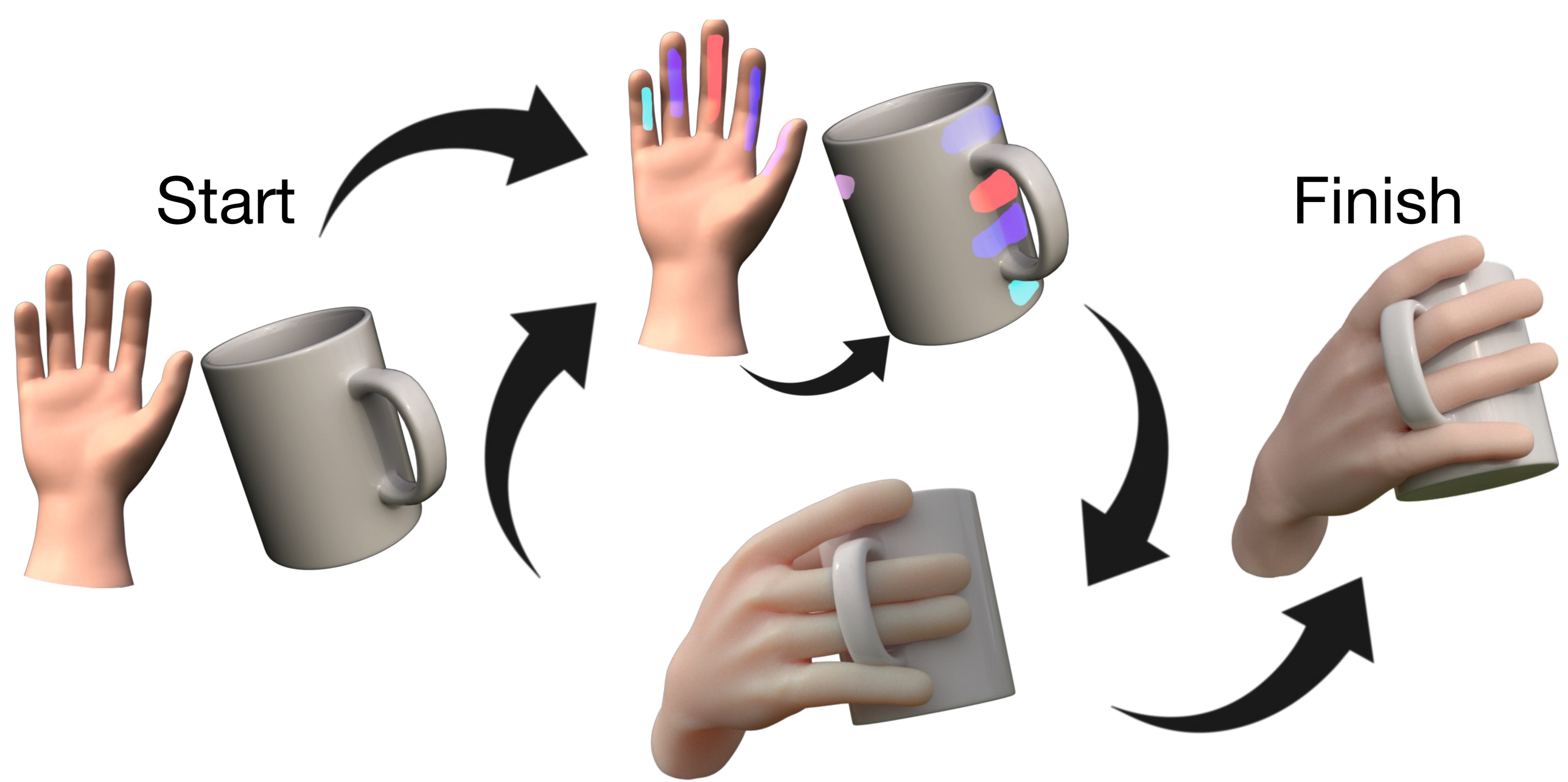
SPE Model

$$c_i^* = \arg\min_y \quad \|log_x(y) - log_x(c_i)\|_2^2$$
$$\text{s.t.} \quad log_x(\cdot) = f(x;\vec{x})$$

$$exp_q(p) := T_q(r_p, \theta_p)$$

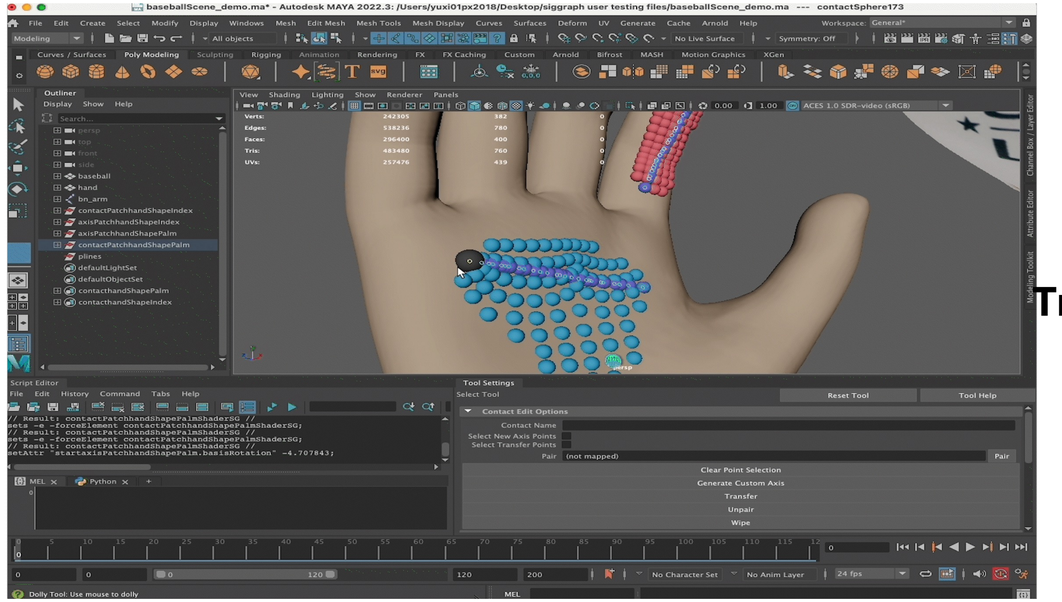
AE Model

What can we do with this model?


Start
Finish
Create and Refine

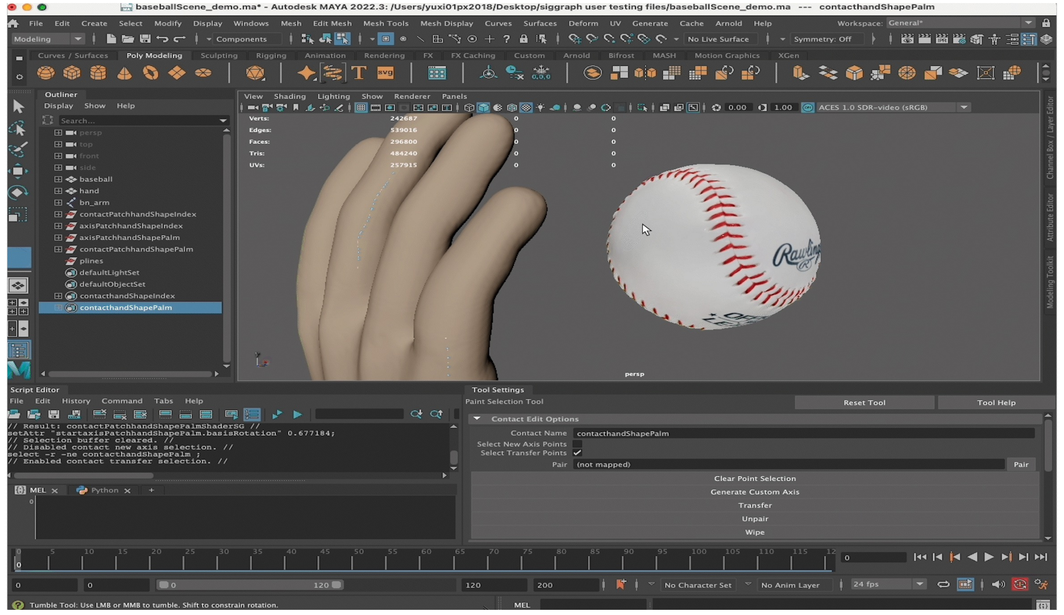
Operation Interfaces

Rotation

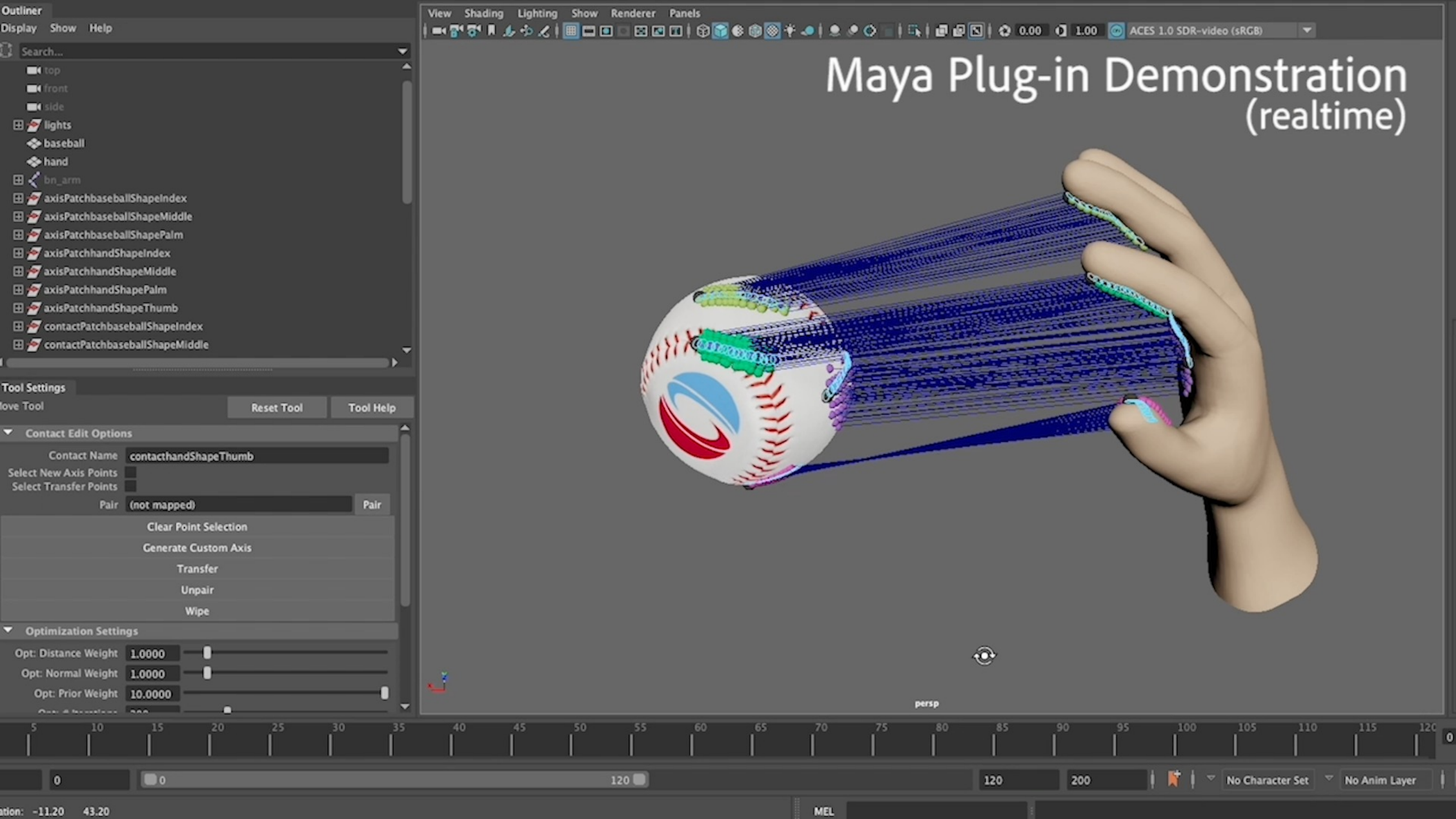


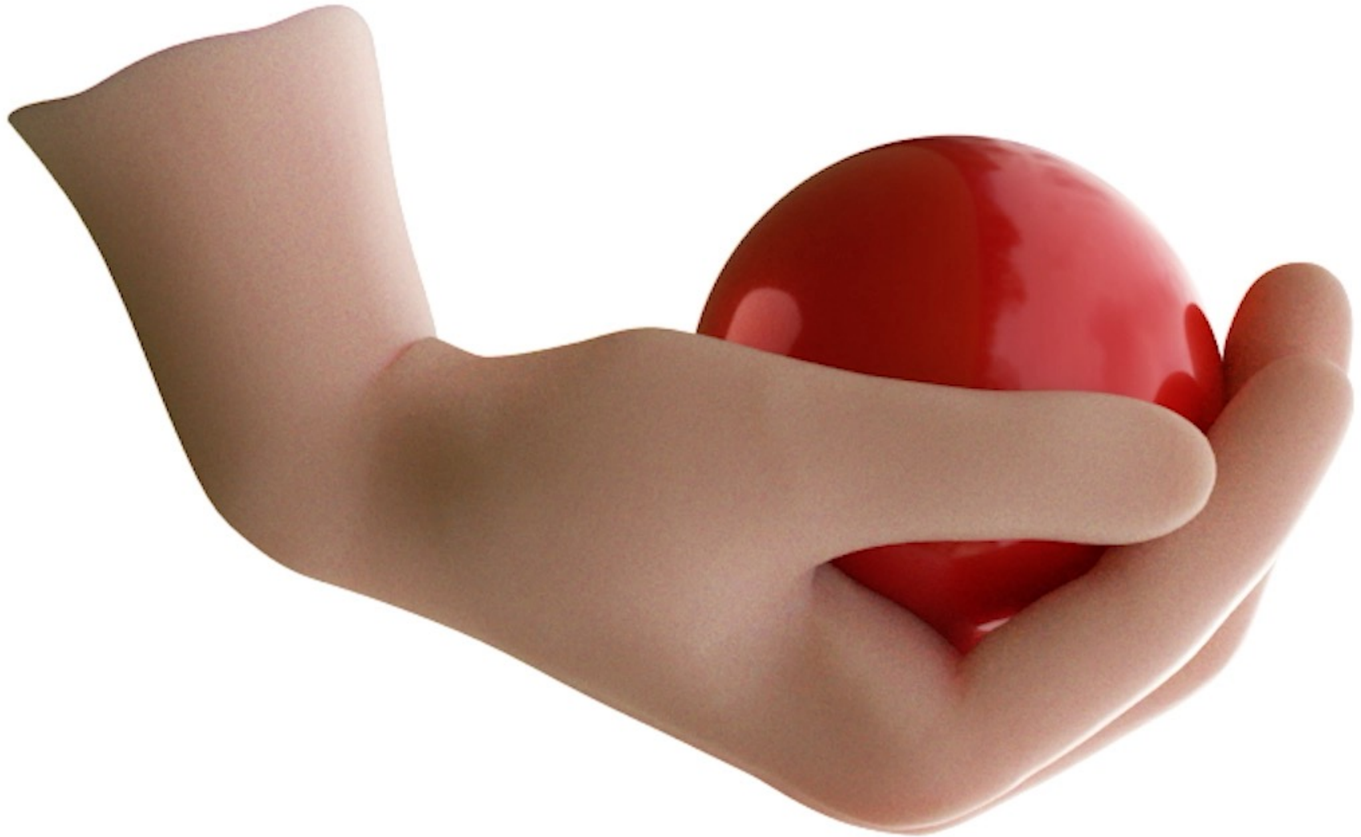
Translation

Isometric Deformation



Transfer


Outliner
Display Show Help
Search...
top
front
side
lights
baseball
hand
bn_arm
axisPatchbaseballShapeIndex
axisPatchbaseballShapeMiddle
axisPatchbaseballShapePalm
axisPatchhandShapeIndex
axisPatchhandShapeMiddle
axisPatchhandShapePalm
axisPatchhandShapeThumb
contactPatchbaseballShapeIndex
contactPatchbaseballShapeMiddle
Tool Settings
Reset Tool
Tool Help
Contact Edit Options
Contact Name
contacthandShapeThumb
Select New Axis Points
Select Transfer Points
Pair
(not mapped)
Pair
Clear Point Selection
Generate Custom Axis
Transfer
Unpair
Wipe
Optimization Settings
Opt: Distance Weight
1.0000
Opt: Normal Weight
1.0000
Opt: Prior Weight
10.0000
View Shading Lighting Show Renderer Panels
ACES 1.0 SDR-video (sRGB)
Maya Plug-in Demonstration
(realtime)
persp
No Character Set
No Anim Layer
MEL




OFFICIAL

Even at Internet Scale!

PICO: 3D People In Contact with Objects from a single image (submitting to CVPR 2025)

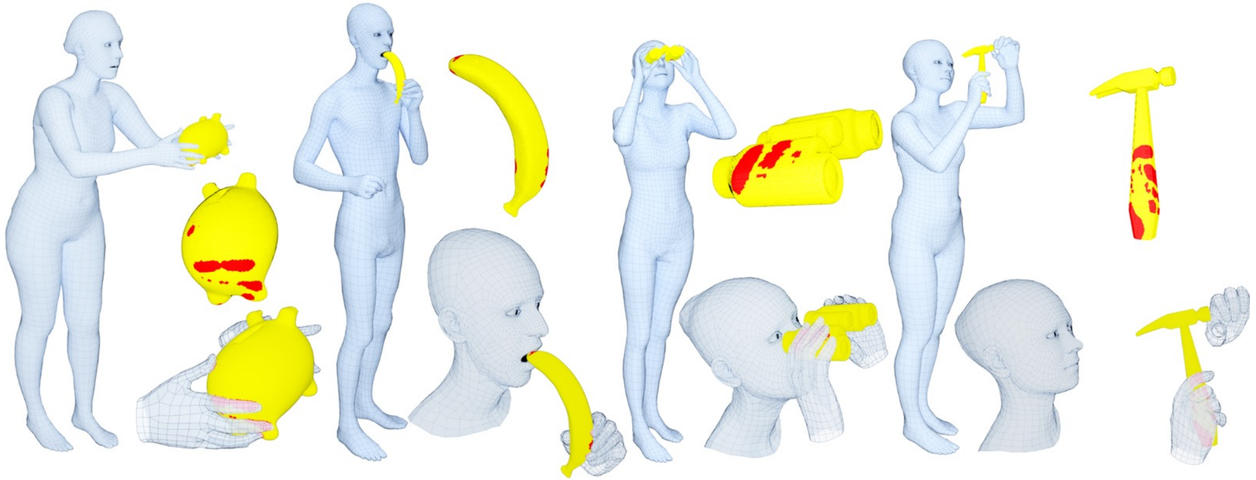
In Collaboration With:

MAX PLANCK INSTITUTE FOR INTELLIGENT SYSTEMS

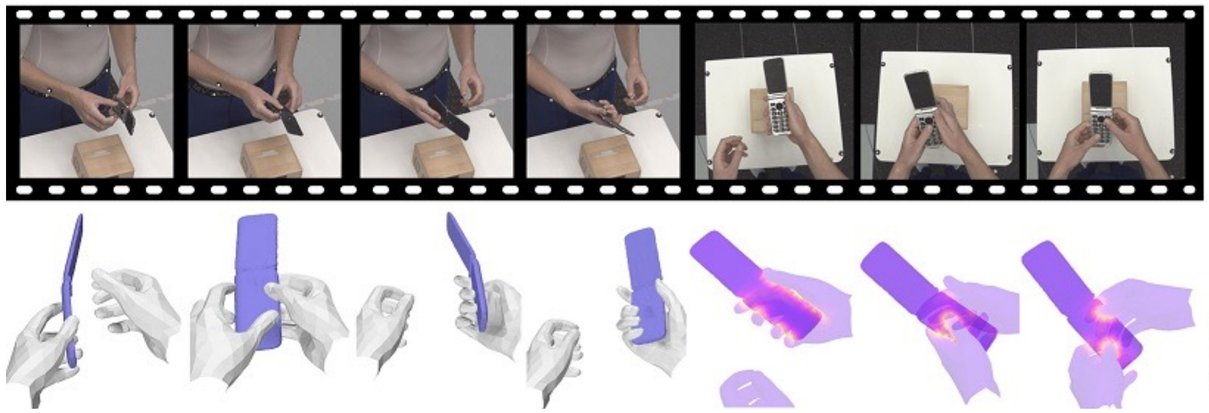
What about motion?

Why is Motion Difficult?

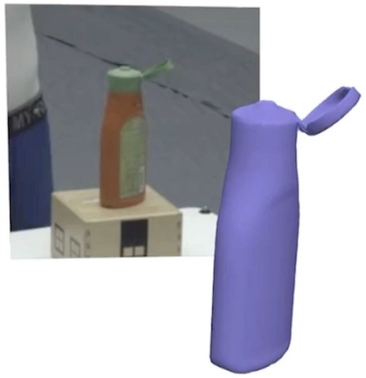
A Silver Lining

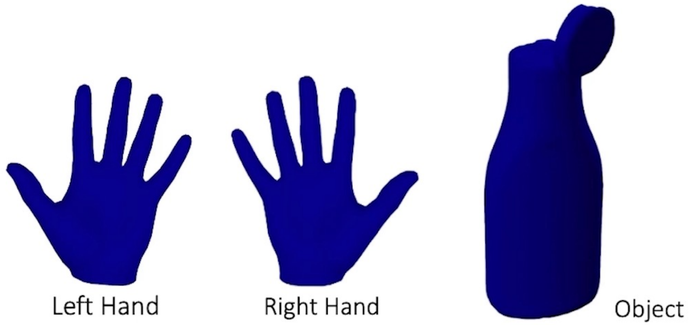


[Taheri et. al, 2020]



[Fan et. al, 2023]

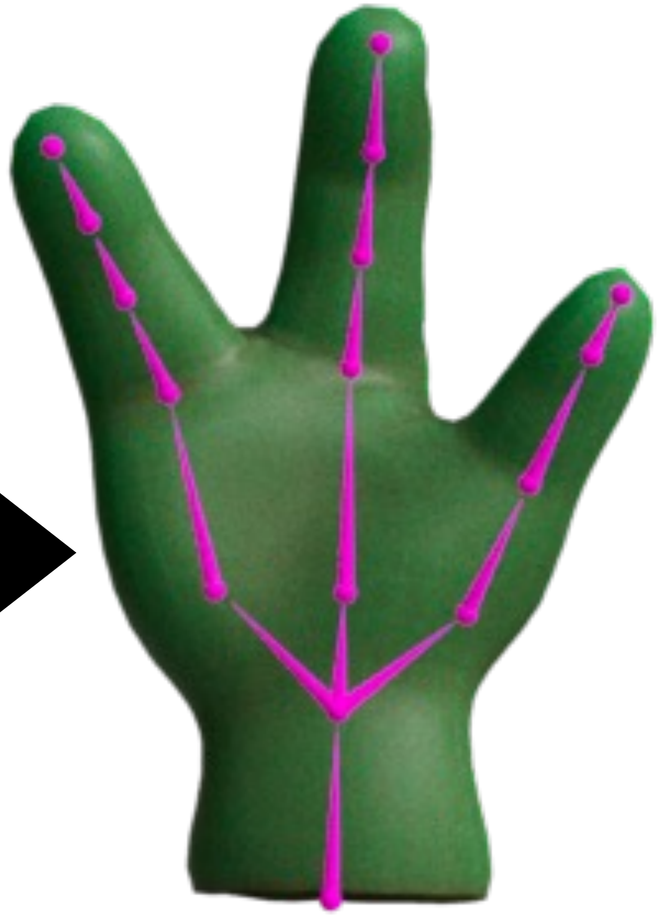




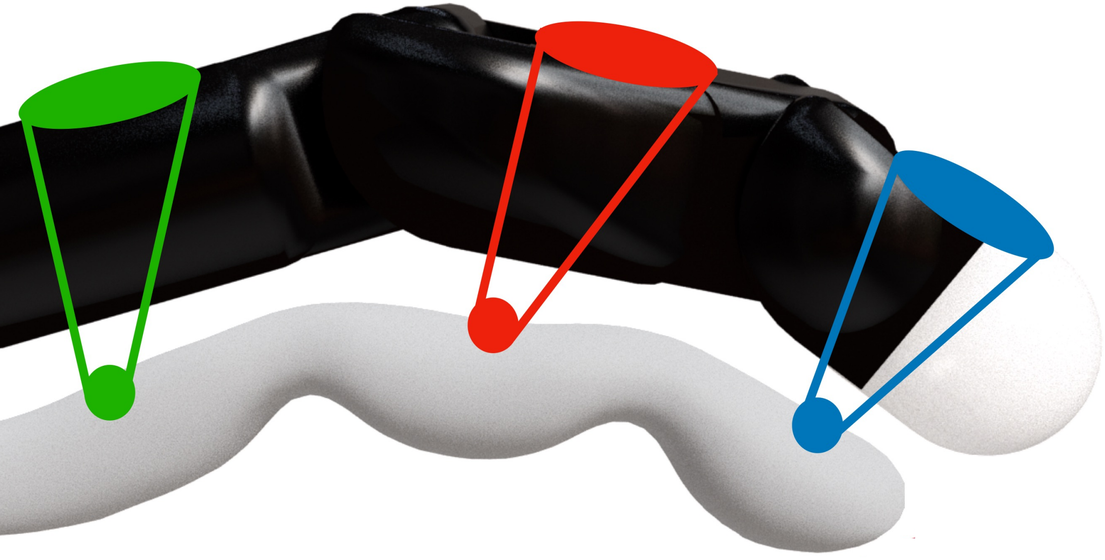
Hand-Object Distance (Brighter: closer)

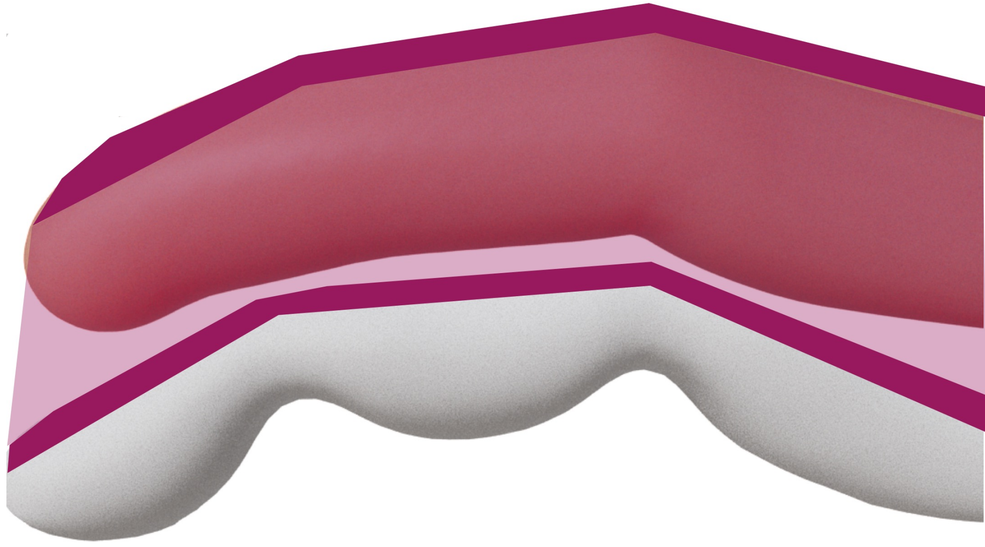
The (Unavoidable) Retargeting Problem

???

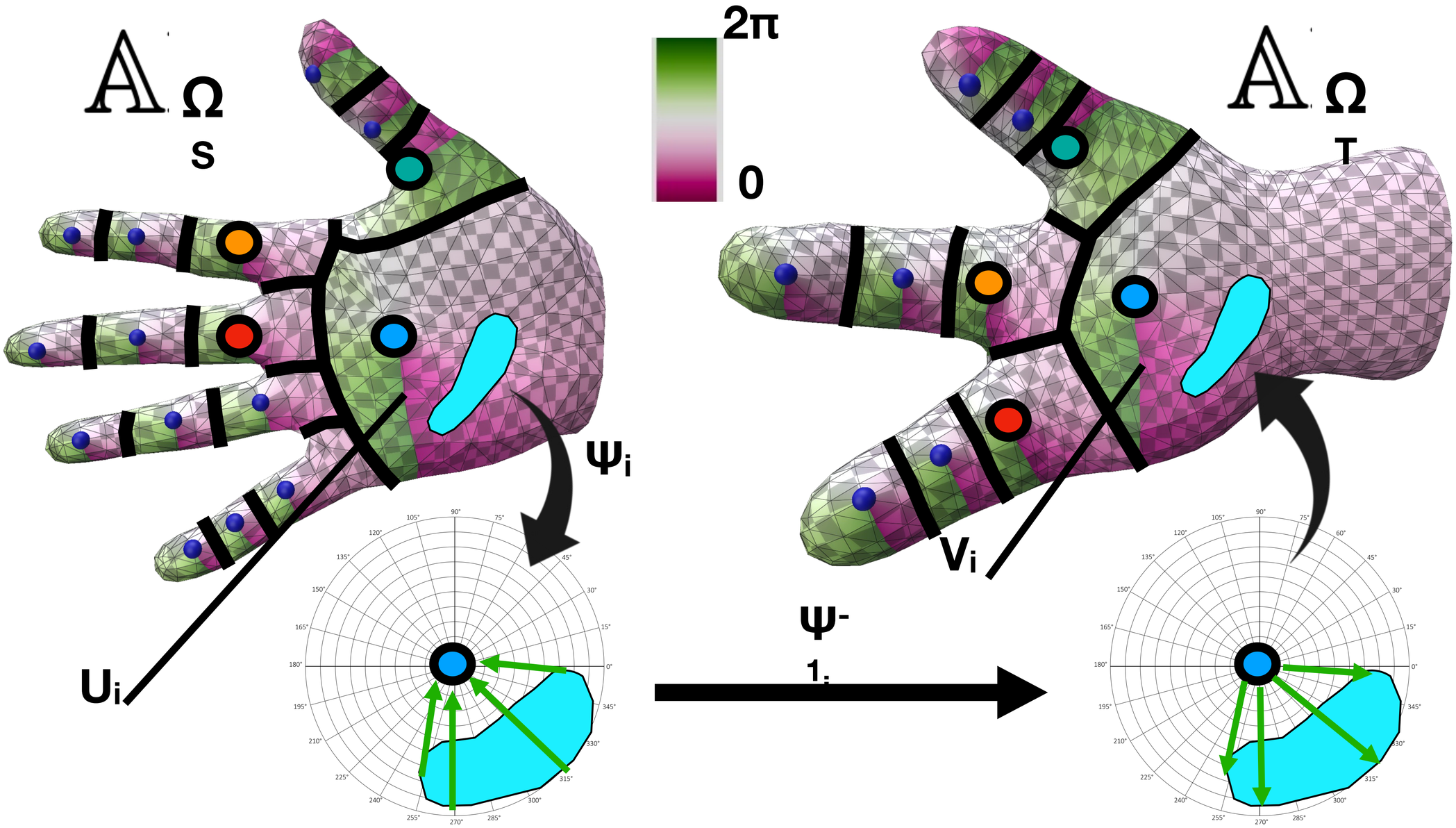


Virtual Markers

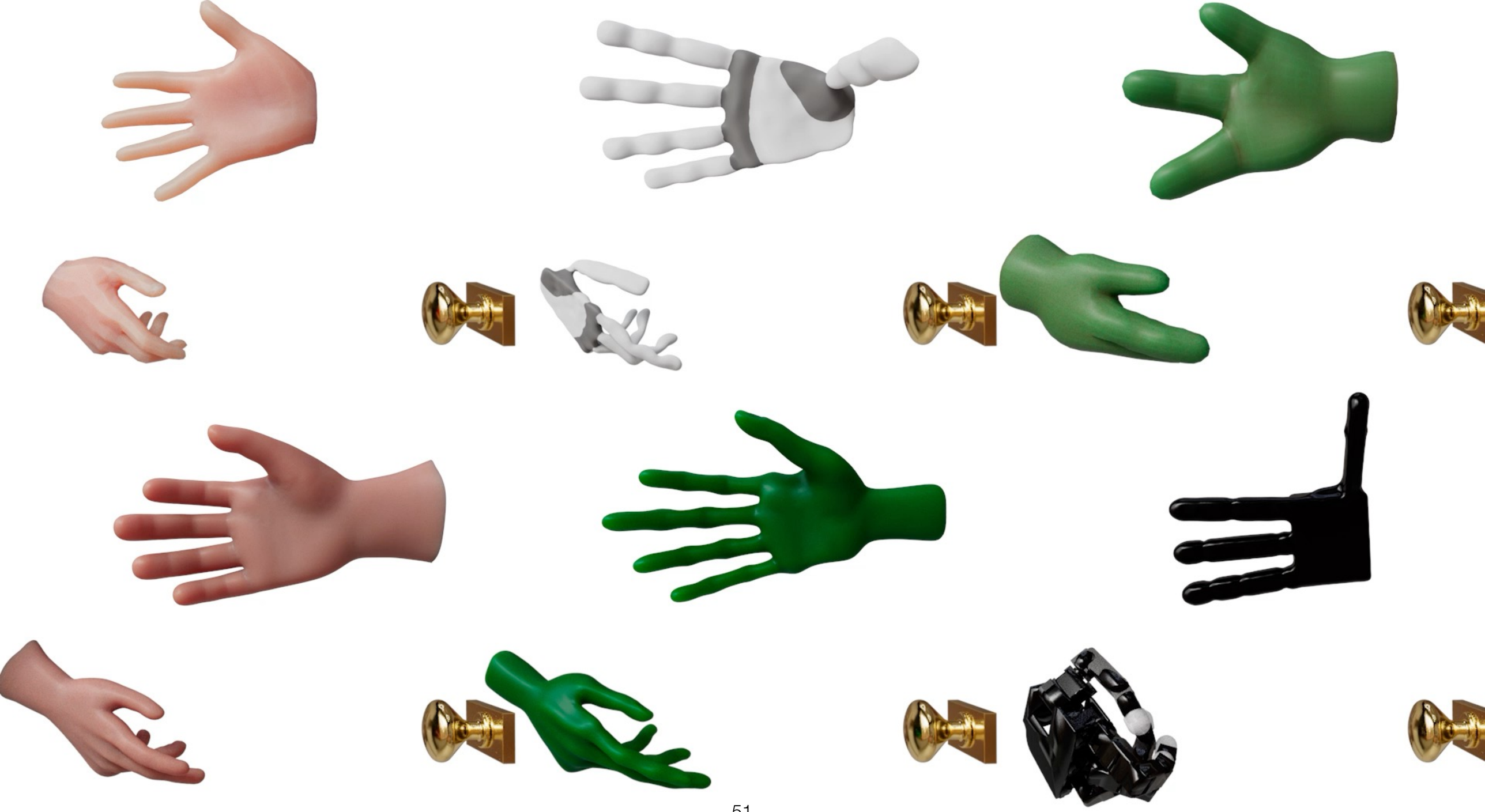


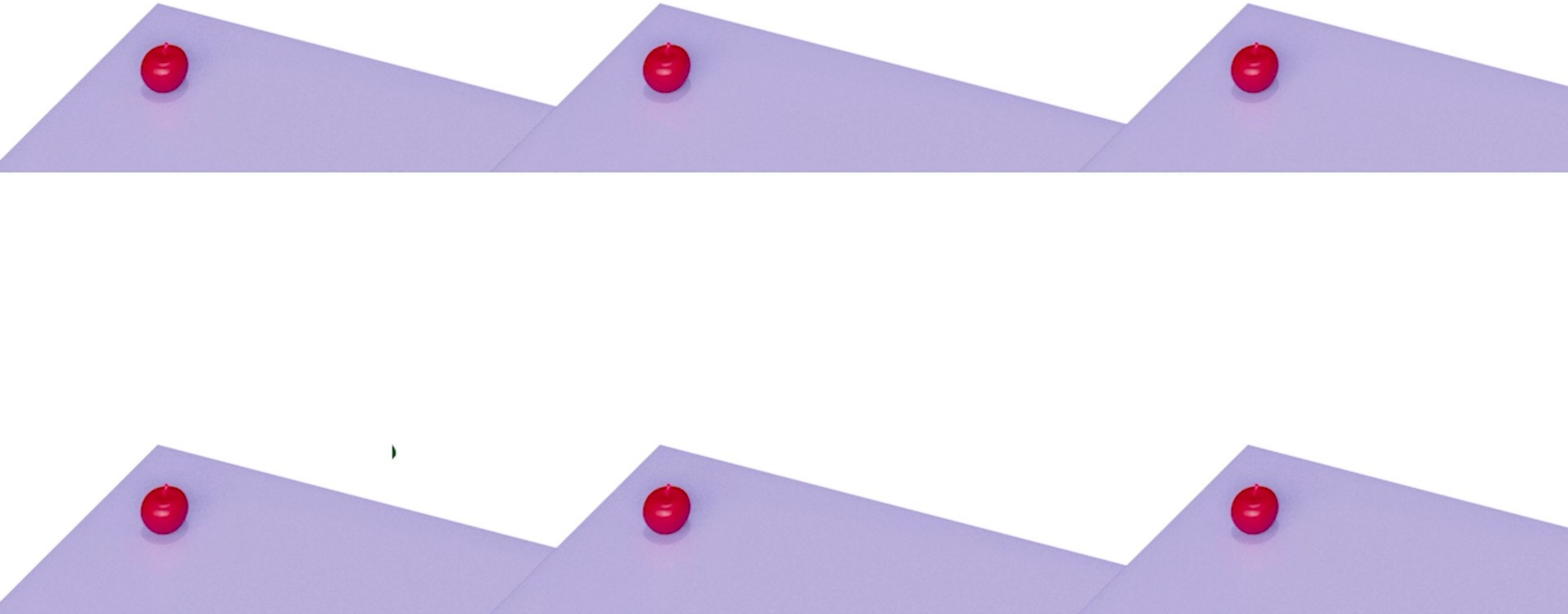


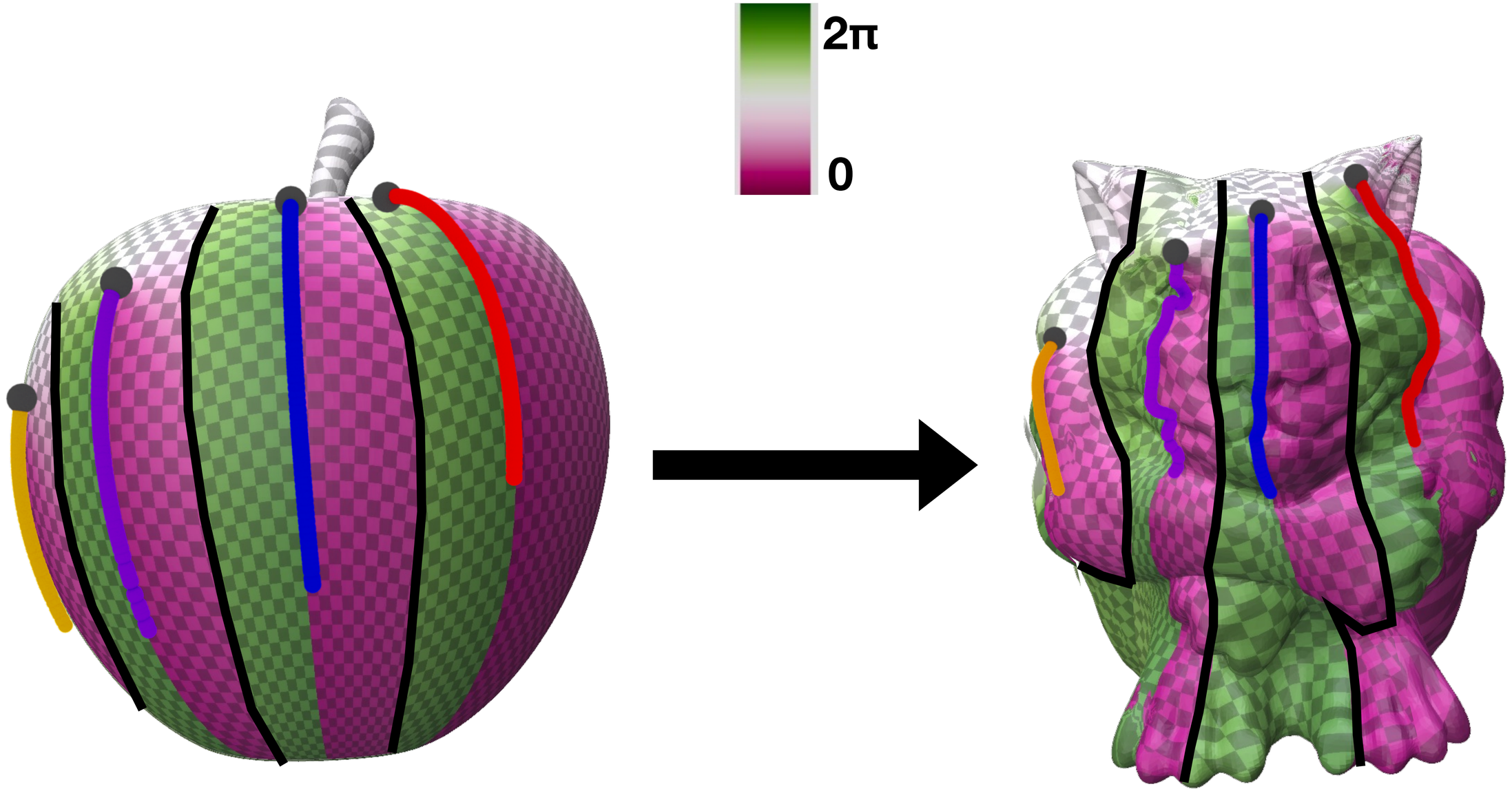
The Contact Atlas



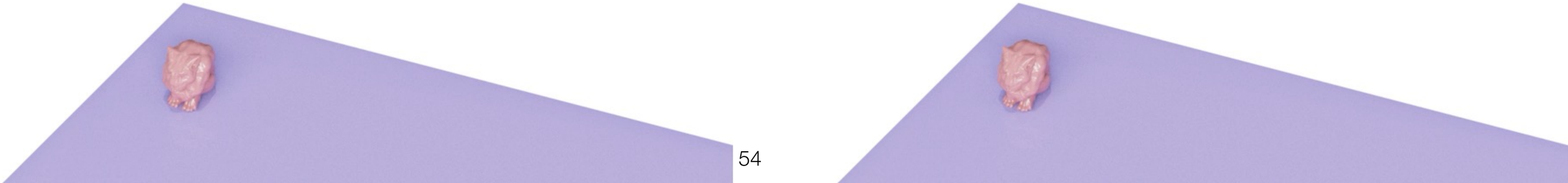
Step 1:
Estimate Initial Trajectory







Index Middle Ring Pinky



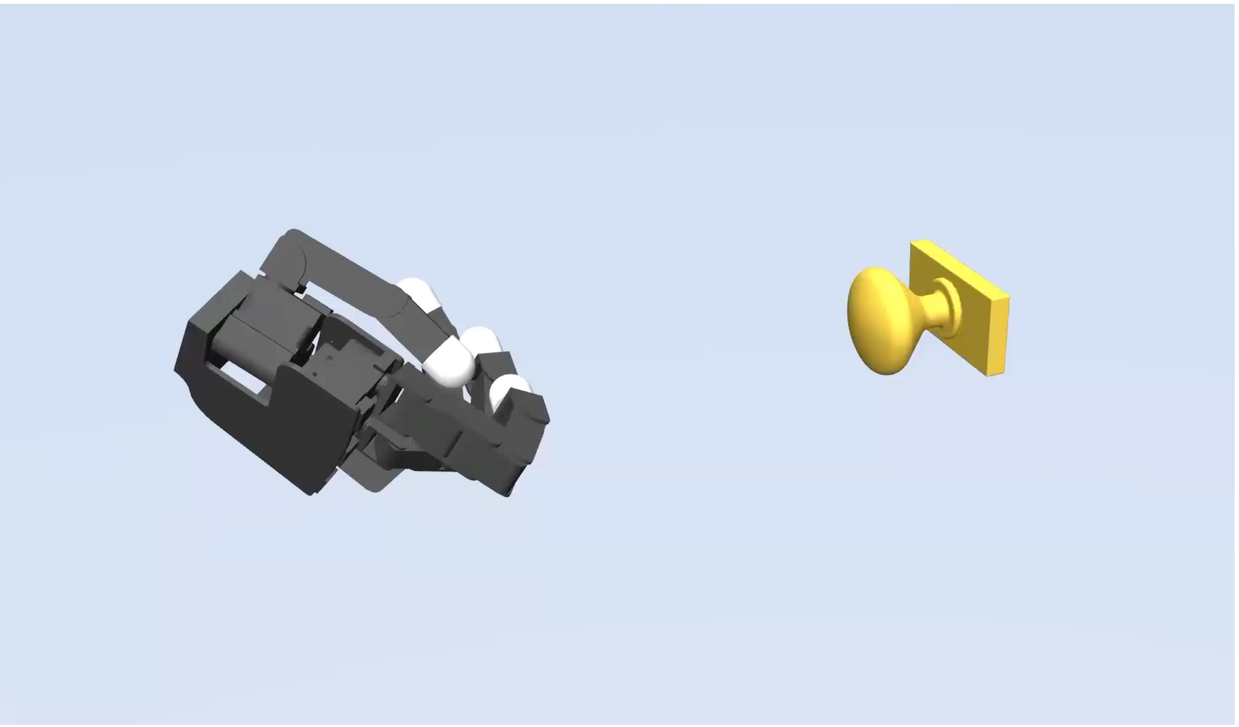
What about physics?

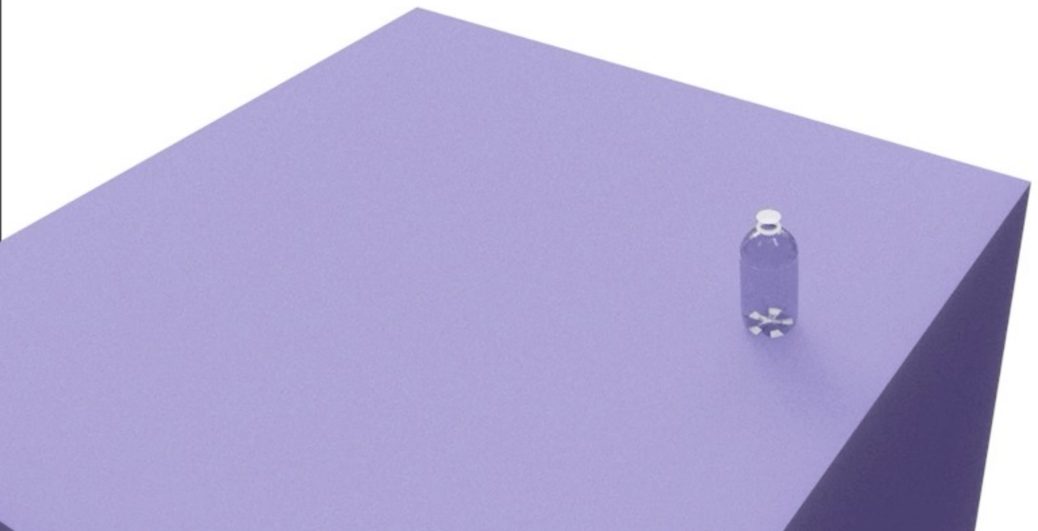
Limitations of Kinematics

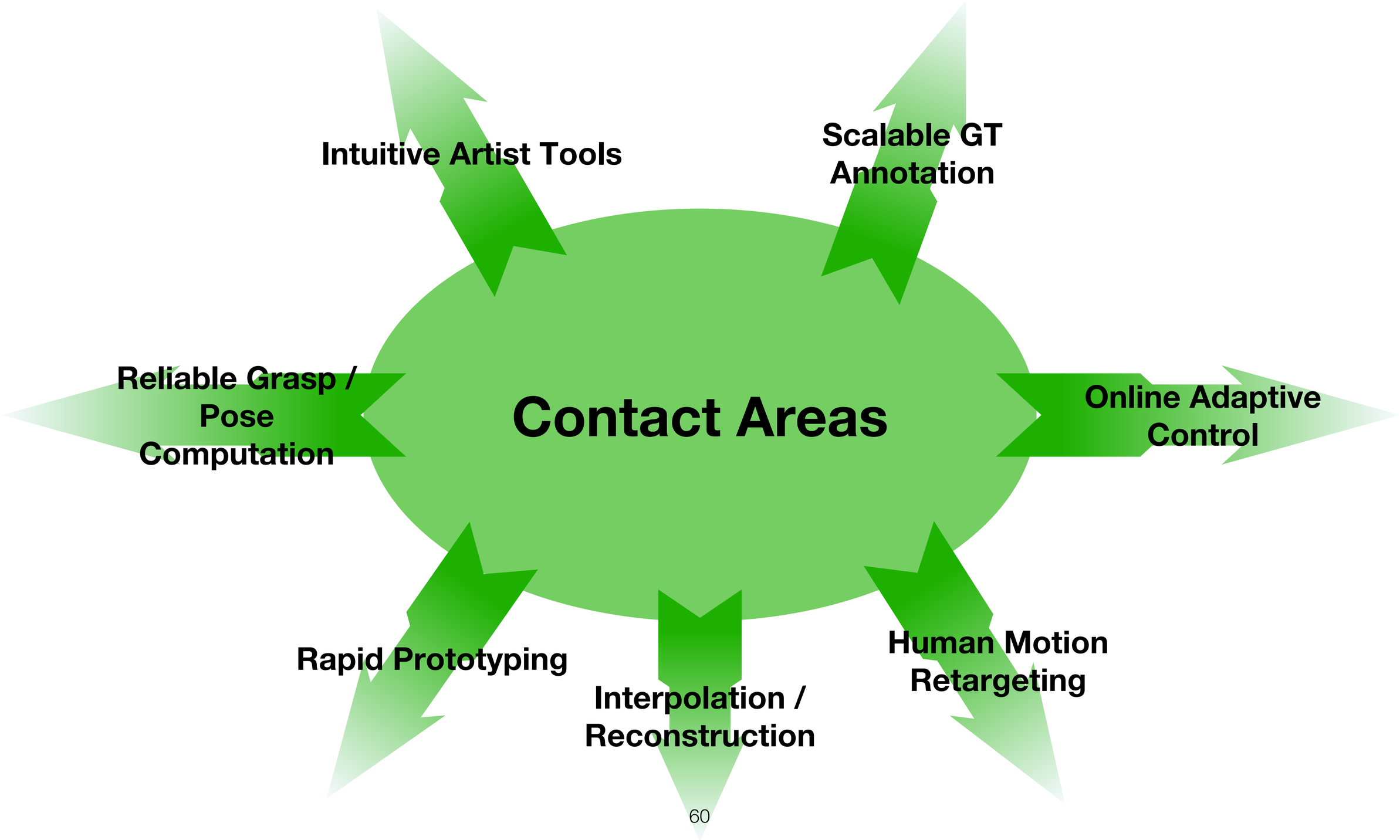
Kinematics Only

Are We Really *That* Far Off?

Direct Kinematic Rollout






Intuitive Artist Tools
Scalable GT Annotation
Reliable Grasp / Pose Computation
Contact Areas
Online Adaptive Control
Rapid Prototyping
Interpolation / Reconstruction
Human Motion Retargeting

Thank You! Questions?