Math and Debugging Review

- Introduction
- Linear Algebra Review
- Debugging Demo
- Q&A

Course Staff

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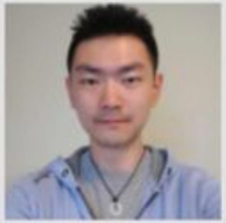

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Inner Product & Dot Product

Inner product maps vectors to scalars and tells us how much vectors "line up"

Standard Euclidean dot product is called the dot product

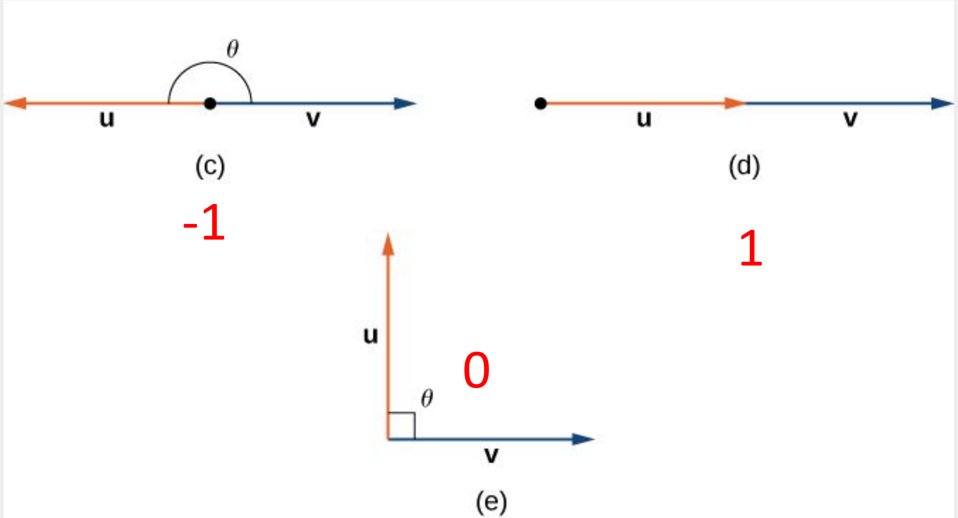
Mathematically:

$$\sum_{i=1}^{n} a_i b_i = a \cdot b = |a||b| \cos\theta$$

Dot Product

Visually:

(assuming these are unit length)



Dot Product

Example:

Which unit-length vector is closer in alignment to the vector V = (1, 0)?

A = (3/5, 4/5) or B = (12/13, 5/13)

Norms

Norm also maps vectors to scalars and tells us the “size” of the vector

For a vector **u**, denoted as ||**u**||

Standard Euclidean norm is ||**u**|| = $\sqrt{\mathbf{u} \cdot \mathbf{u}}$

“Normalize” a vector (make it unit length) with **u** / ||**u**||

Cross Product

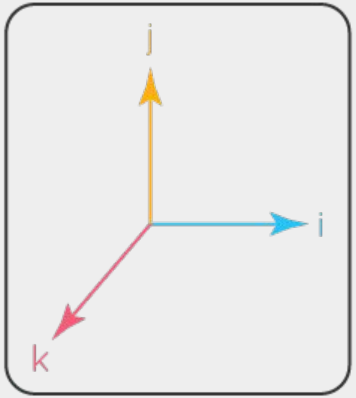
Mathematically:

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \qquad \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = \vec{a} \times \vec{b}$$

$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\vec{c} = \hat{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \hat{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \hat{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$



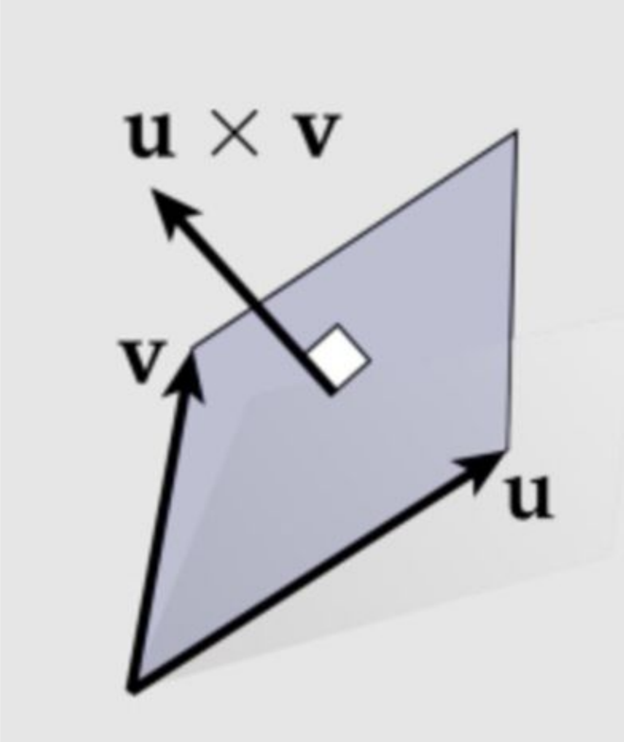
$$\vec{c} = \hat{i}\left|a_2b_3 - a_3b_2\right| - \hat{j}\left|a_1b_3 - a_3b_1\right| + \hat{k}\left|a_1b_2 - a_2b_1\right|$$

Cross Product

Visually:

- The magnitude of the vector is equal to the area of the parallelogram formed by u and v

- The vector itself points perpendicular to the other two vectors (right hand rule!)



2D Cross Product

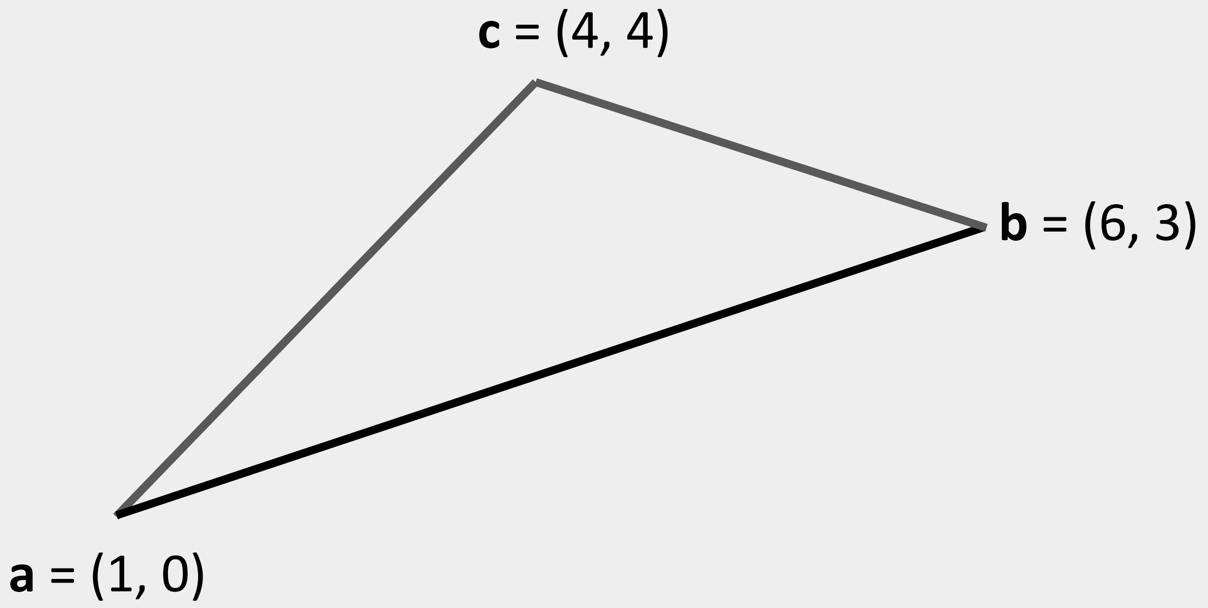
- Technically, a 2D cross product does not exist, but we can abuse notation and get:

$$\mathbf{u} \times \mathbf{v} := u_1 v_2 - u_2 v_1$$

- We can also say that the area of a triangle formed by 2 vectors is half of the cross product (since it would be half of the parallelogram)

Cross Product

Example: What is the area of this triangle?



Matrices

Always ask "*What does this matrix represent?*"

Matrices can be a lot of things

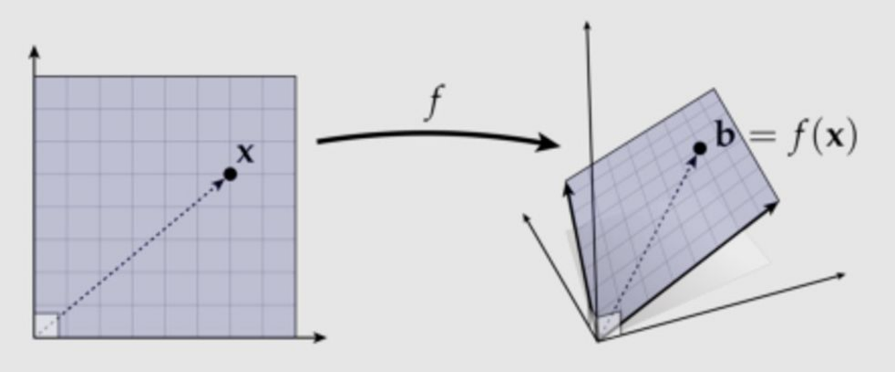
- Linear maps
- Linear systems of equations
- Graph adjacency matrix
- Kernels for image convolution
- etc!

$$\begin{bmatrix} 1 & 7 & 3 \\ 4 & 9 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

Matrices as Linear Maps

A linear map

1. maps lines to lines and preserves the origin
2. preserves vector space operations (add/scale)



So a linear map f: $\mathbb{R}^n \rightarrow \mathbb{R}^m$ can be expressed as a *m x n* matrix

Matrices as Linear Maps

Example: Compute the matrix $A \in \mathbb{R}^{3\times 2}$ representing the linear map

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}^3; \qquad (x_1, x_2) \rightarrow (4x_1, x_2 - x_1, -2x_2)$$

Composing Linear Maps

Example: Compute the matrix $A \in \mathbb{R}^{2\times2}$ representing the linear map

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}^2; \qquad (x_1, x_2) \rightarrow (3x_1, 2x_2)$$

Example: Compute the matrix $B \in \mathbb{R}^{2\times2}$ representing the linear map

$$g: \mathbb{R}^2 \rightarrow \mathbb{R}^2; \qquad (x_1, x_2) \rightarrow (x_1, -x_2)$$

Example: Compute the matrix $C \in \mathbb{R}^{2\times2}$ representing the linear map $f \circ g$

Derivatives & Integration

Derivatives represent rate of change of a function

-Animation (Tangents on splines and IK gradient descent)

-Particle simulation (Understanding velocity and acceleration)

Integrals represent a continuous summation over some function

- Rendering Equation (Summation of light over hemisphere)
- Variance Reduction (Summation of error between render and average)
- CDFs for Importance Sampling (Summing probabilities based on luminance)

Jacobian Matrices

Collects all first-order partial derivatives of a multivariate function

$$\mathbf{J} = \frac{d\mathbf{f}(\mathbf{x})}{d\mathbf{x}} = \left[\frac{\partial \mathbf{f}(\mathbf{x})}{\partial x_1} \cdots \frac{\partial \mathbf{f}(\mathbf{x})}{\partial x_u} \right] = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_u} \\ \vdots & & \vdots \\ \frac{\partial f_v(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial f_v(\mathbf{x})}{\partial x_u} \end{bmatrix}$$

Jacobian Matrices

Example: Let $x_1 = f_1(u, v) = u^2 - v^2$, and $x_2 = f_1(u, v) = 2uv$

Find the jacobian J(u, v).

Polar/Spherical Coordinates

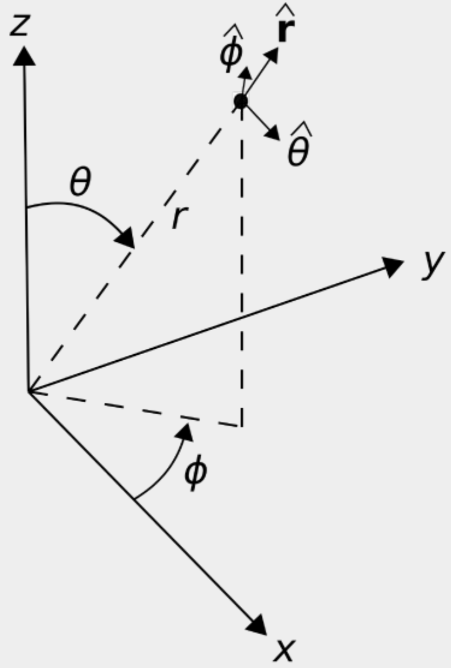
Spherical coordinates are an extension of polar coordinates

We can convert between the spherical point $P\ (r, \phi, \theta)$ and the cartesian point $P\ (x, y, z)$

$$x = r \cos \phi \sin \theta \qquad r = \sqrt{x^2 + y^2 + z^2}$$

$$y = r \sin \phi \sin \theta \qquad \theta = \arccos(z/r)$$

$$z = r \cos \theta \qquad \phi = \arctan(y/x)$$



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Debugging Demo

Let’s code!

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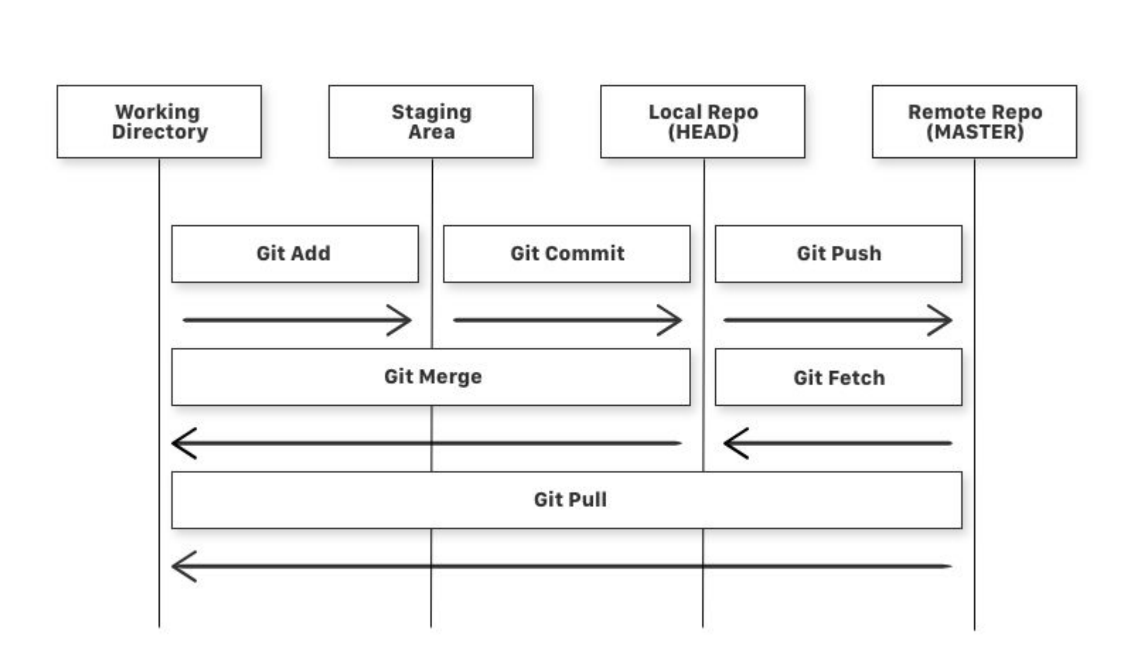
Git Overview

- clone
- add
- commit
- push
- pull
  - merge
  - rebase
  - etc.



Git Final Notes

- **ALWAYS** pull from source repo before starting a new assignment
- Push to your git repo before submitting to Gradescope
  - Wait at least a minute or two in between pushing and submitting
- Check if your submission compiles in Gradescope
  - If you get a negative score, your submission **did not compile**

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